37th International Championship for Mathematical and Logical Games, 26.08.2023

Information and results at http://gmil.pwr.edu.pl/

START for ALL PARTICIPANTS

1. TWO BOARDS (coefficient 1)

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Pieces are removed from these boards until they appear to be identical.

How many pieces will then have been removed, at least?

2. CIRCLES AND LINES (coefficient 2)



If we draw a circle and two straight lines, we obtain a maximum of 5 points of intersection.

How many will be obtained, at most, by drawing 2 distinct circles and 3 distinct straight lines?

3. DIFFERENCE OF 2 or 3 (coefficient 3)

Matthew wants to place the numbers from 1 to 9 (1 and 9 are already placed) in the discs of the figure so that the difference between two neighbouring numbers (the larger minus the smaller) is always equal to 2 or 3.



What number will go in the shaded box?

4. FROM 1 TO 23 (coefficient 4)



By using one or more of these six tokens, Matthew can form different numbers, for example:



How many numbers, between 1 and 23, will he not be able to form? Note: Only one of each token is available.

5. LABYRINTH (coefficient 5)



In this maze, you enter through the box located at the top left and exit through the box located at the bottom right by going through the seven letters of the word WROCŁAW in order. You can only move in the direction of the arrows.

How many different paths are there?

END for CE PARTICIPANTS

6. SOME SUMS OF SUMS (coefficient 6)

In the square boxes of a diagram of 3 boxes by 3, place all the numbers from 1 to 9. The four circles will contain the sums of the numbers in the 4 squares that they



each partially cover. Finally, take the sum of the four numbers in the circles.

What is the maximum value of this last sum?

7. WATER BOTTLES (coefficient 7)

The organizers of the International Final decide to fill water bottles for the participants and accompanying persons. They have 350 bottles to fill at two taps: at one of the taps, they can fill three bottles in two minutes and at the other they can fill two bottles in one minute.

What is the minimum time in hours and minutes needed to fill the 350 bottles?

8. COMPLETED RECTANGLE (coefficient 8)



From a rectangle of 2 squares by 3, we create a new figure by adding a square on each side of the rectangle, this square having a side coinciding with a side of a square of the rectangle.

How many different figures can we make? *Figures that look identical after rotation or flipping only count as one!*

END for CM PARTICIPANTS

Problems 9 to 18: beware! For a problem to be completely solved, you must give both the number of solutions, AND give the solution if there is only one, or give any two correct solutions if there are more than one. For all problems that may have more than one solution, there is space for two answers on the answer sheet (but there may still be just one solution).

9. A VERY SPECIAL NUMBER (coefficient 9) A three-digit number, with all-different non-zero digits, is the sum of all two-digit numbers formed from two of the three digits of the initial number. What is this number?

10. BIRTHDAY GIFT (coefficient 10)

For his birthday, Matthew has just received a book, a collection of mathematical games whose pages are numbered from 1, in order and without skipping numbers. Matthew has fun counting the total number of digits used for page numbering. He finds that this number is equal to 2.5 times the number of numbered pages.

What is the last page number?

11. A RELATIVELY LARGE NUMBER (coefficient 11)

Daniel enjoys doing exercises in arithmetic. He divides each three-digit number (all digits nonzero) by the product of its digits. For some of these numbers the result of the division is an integer. What surprised him was when for a certain number the quotient turned out to be whole and greater than 50.

What is this three-digit number?

END for C1 PARTICIPANTS

12. CRYPTARITHM (coefficient 12)

In this cryptarithm the same letter always replaces the same digit and the same digit is always replaced by the same letter.

$POLAND \times 3 = WROCLAW$

We specify that L = 0. What is the value of WROCLAW?

13. CONSTELLATION (coefficient 13)



The numbers from 1 to 7 are placed in the seven stars of the constellation. Each of the larger stars contains a number greater than those in its two or three neighbours. We then calculate the differences, taken in absolute values, between numbers in two stars connected by a segment, and we add these differences.

What is this sum, at most?

14. STAR OF THE YEAR (coefficient 14)



The discs of this star must contain all the whole numbers from 1 to 19 (the numbers 5, 6 and 19 are already placed) so that:

- the sum of the three numbers written on each of the spokes, starting from the central 6 (which will be counted) is always equal to 23;

- the sum of the three numbers written on each side of the hexagon is also always equal to 23.

What number will go in box a?

END for C2 PARTICIPANTS

15. INCREASING SEQUENCE (coefficient 15)

Zbyszek builds an increasing sequence made up of all the whole numbers that are written only with the digits 3 and 7. The start of his sequence is $3, 7, 33, 37, 73, \ldots$

What number will be the 2023rd in Zbyszek's sequence?

16. COUNT THEMALL (coefficient 16)

Count Themall owns a parcel of land ABC in the shape of a right-angled triangle.



This land is divided into three plots: IDBE, IECF and IFAD, point I being located at the intersection of the bisectors of angles B and C. Each of the straight lines (ID), (IE) and (IF) is perpendicular to a side of the ABC triangle. The IDBE plot has an area of 676 m² and the IECF plot an area of 1014 m².

What is the area of the third plot?

END for L1, GP PARTICIPANTS

17. FIFI'S SEQUENCE (coefficient 17)

Figaro, nicknamed Fifi, has written successive numbers of a sequence as follows:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

where each term is equal to the sum of the two previous terms. He stopped after writing the first number which is a multiple of 100.

How many numbers did he write in total?

18. LOST IN THE FOREST (coefficient 18) Alice, Bertrand and Camilla are lost in a huge forest. Thanks to their respective smartphones, they establish that their three positions form a triangle whose dimensions are all whole numbers of metres and such that one of the angles of the triangle measures exactly 5 times the measurement of another angle. Knowing that Bertrand and Camilla are less than 500 metres from each other, what is the distance between Alice and Camilla?

END for L2, HC PARTICIPANTS