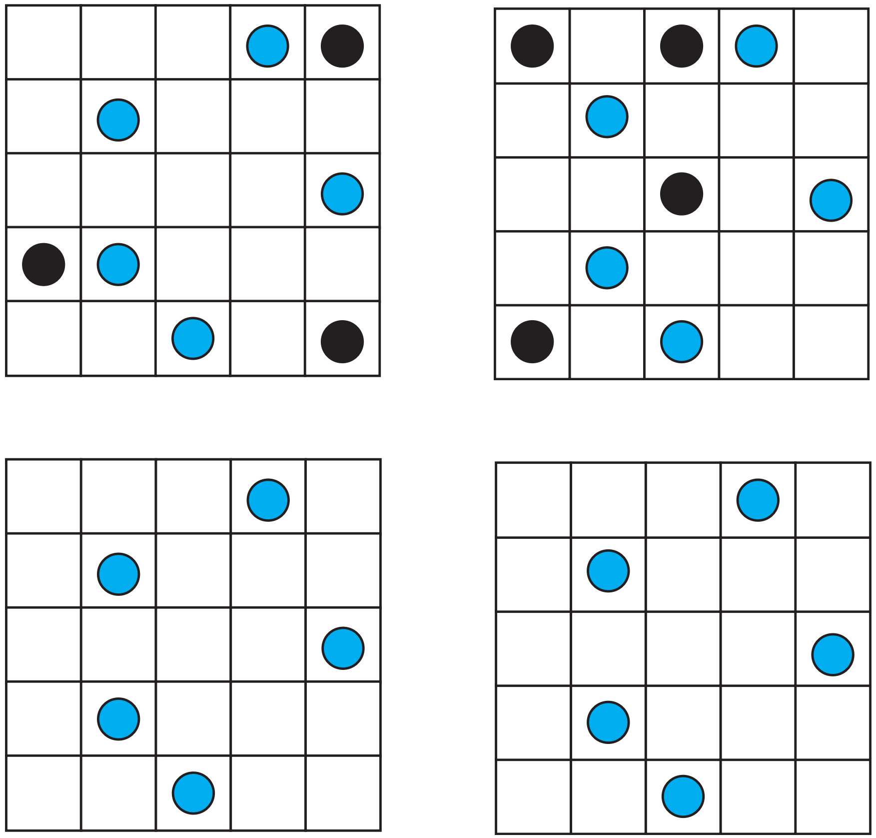
37e Championnat International des Jeux Mathématiques et Logiques

26 august 2023

Solutions (RAMA)

Problem 1



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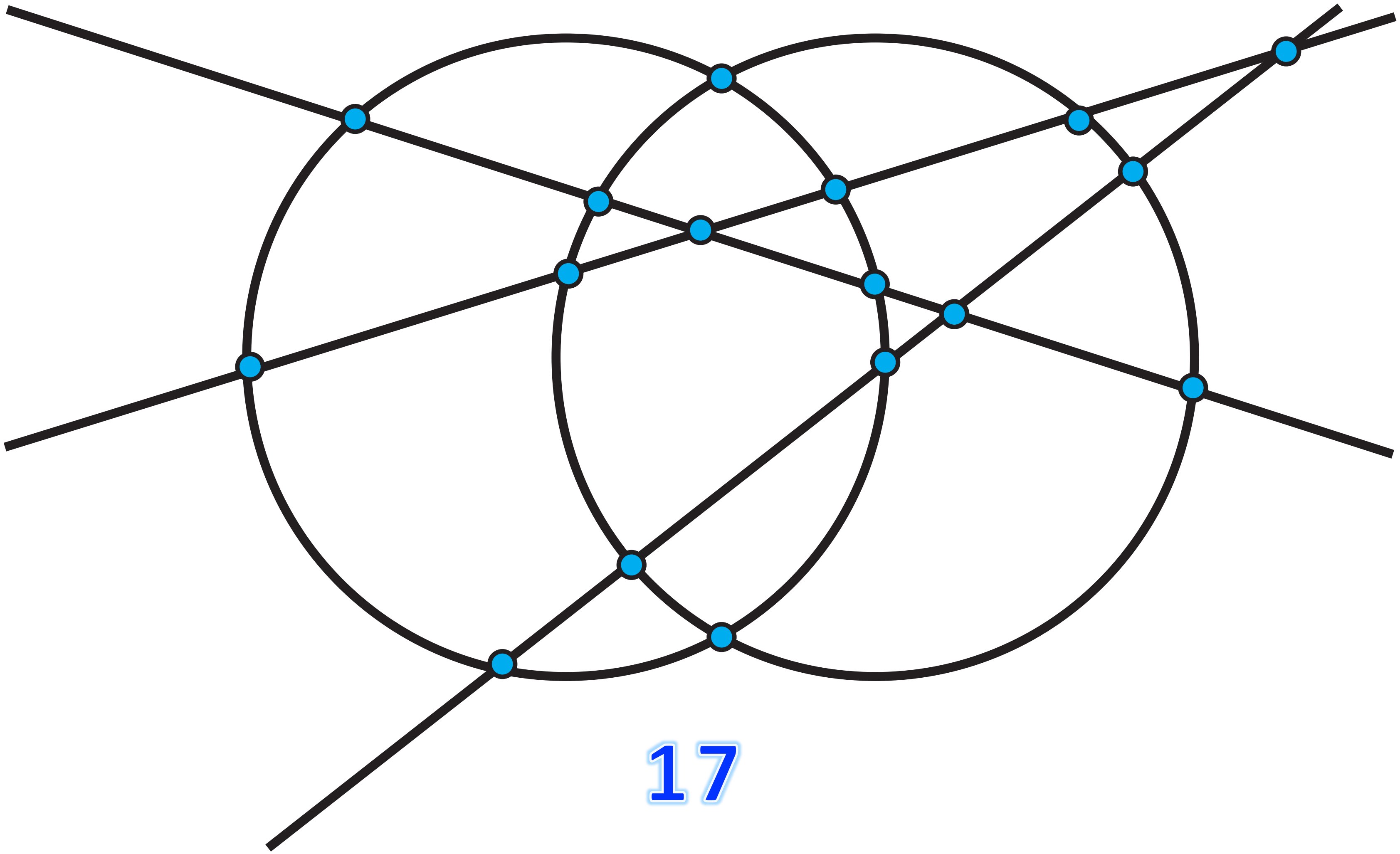
Problem 2



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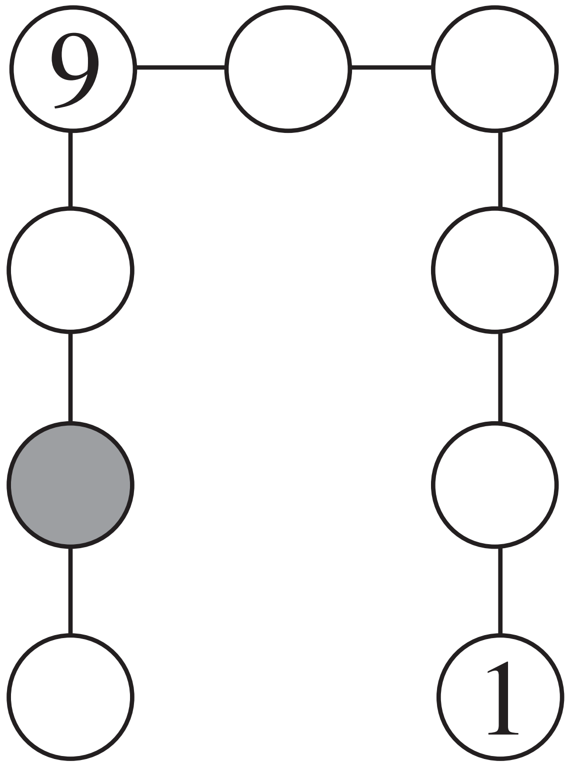
Problem 3



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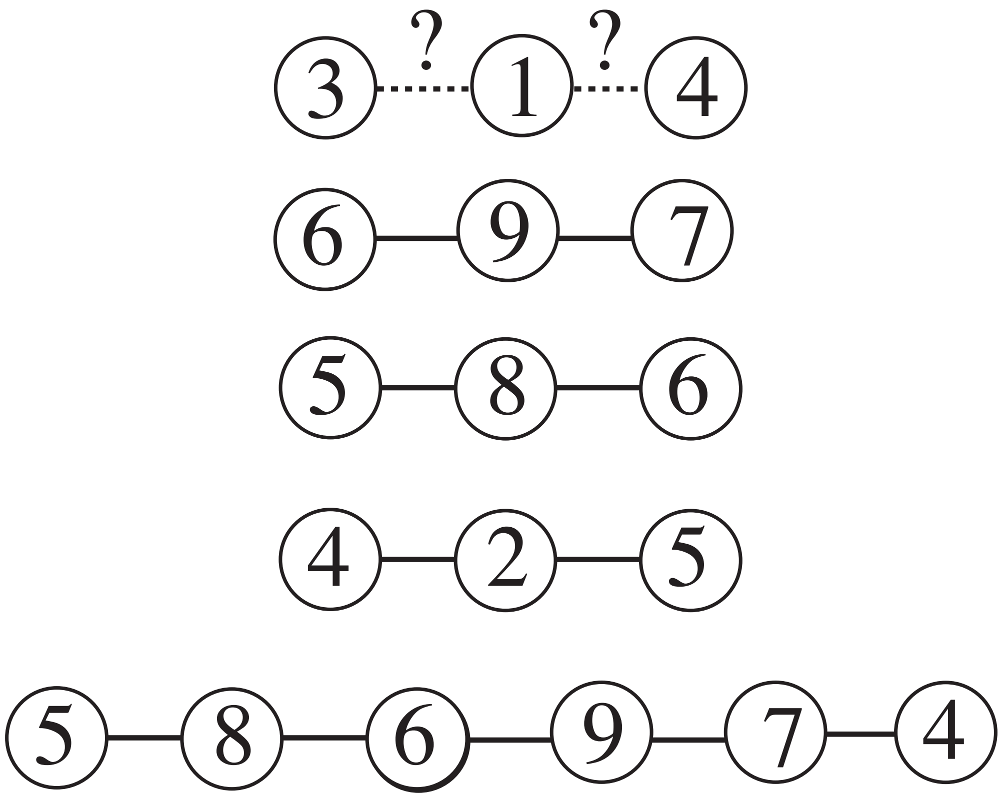
Problem 3



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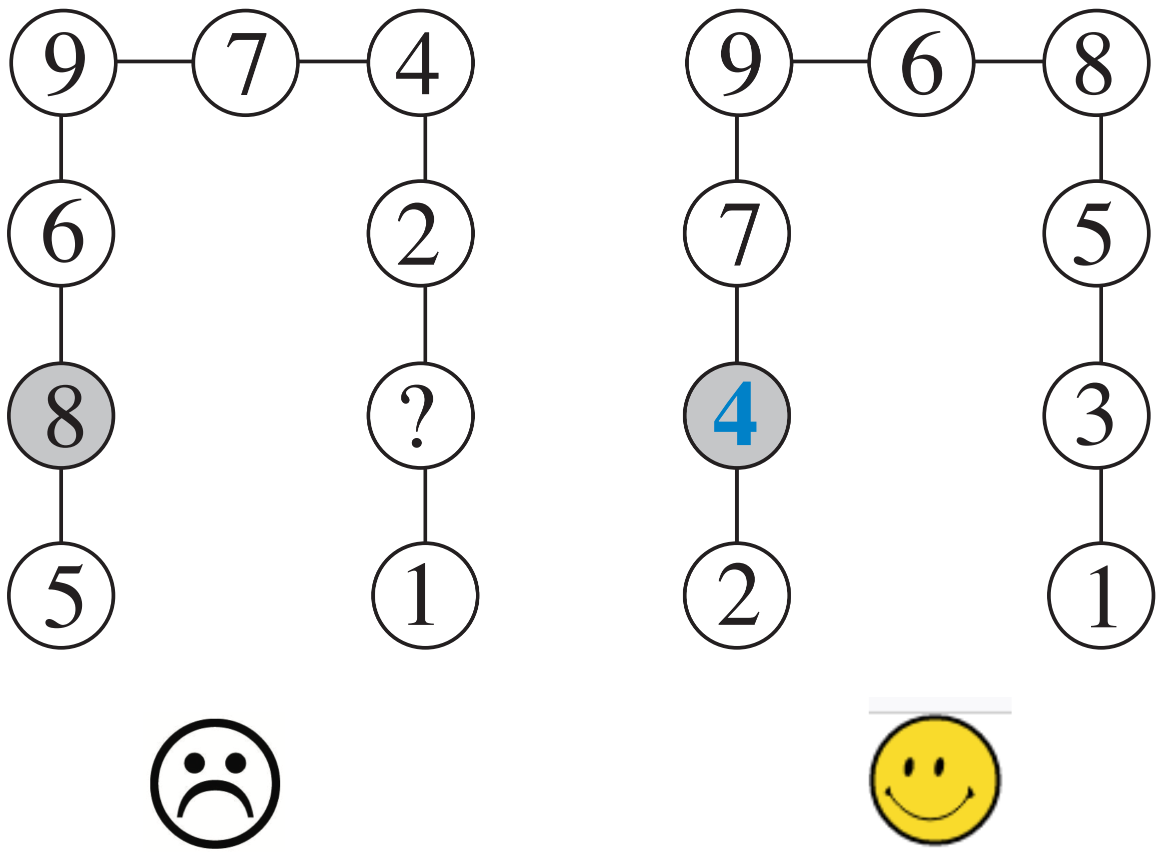
Problem 3



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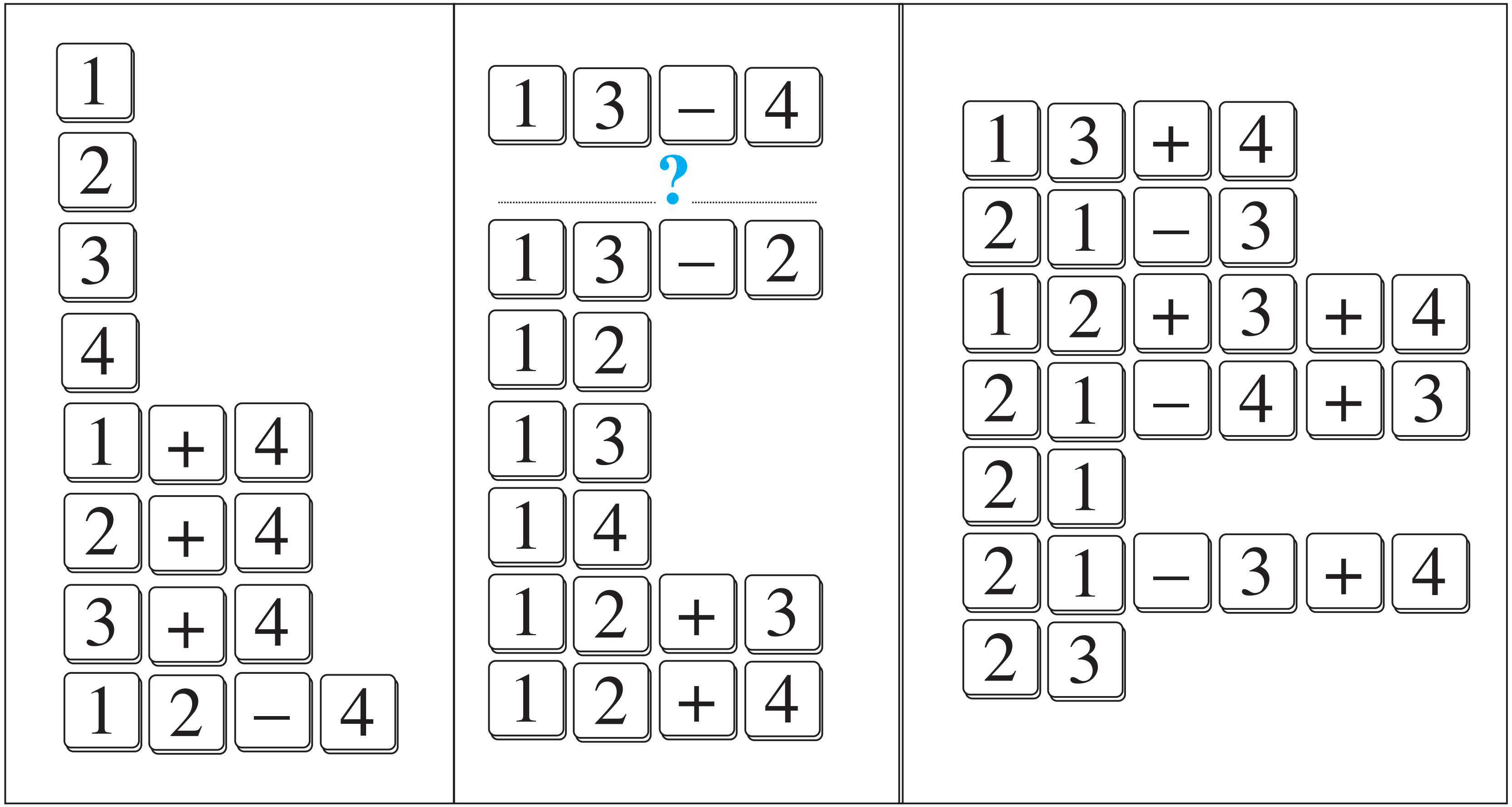
Problem 4



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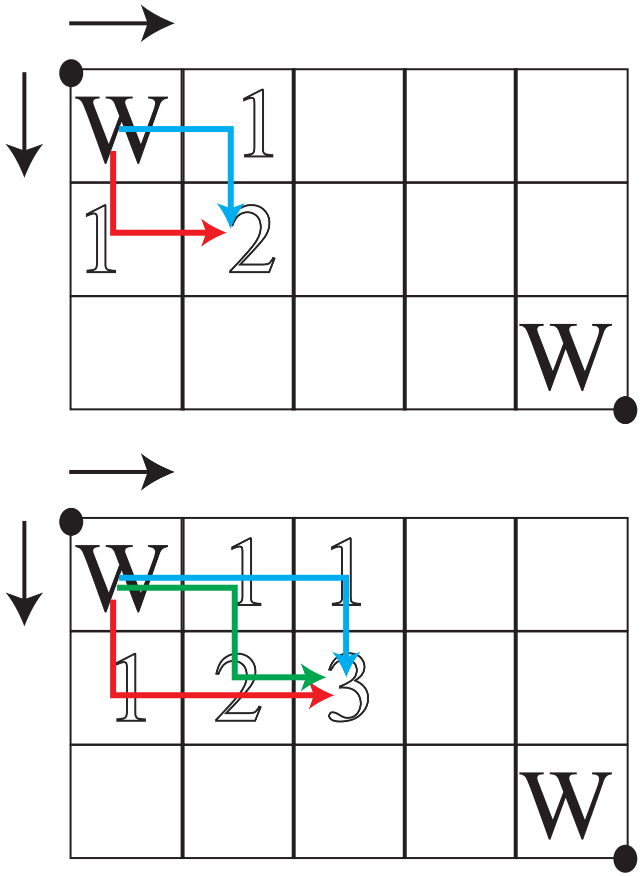
Problem 5



W	1	1	1	1
1	2	3	4	5
1	3	6	10	**15**

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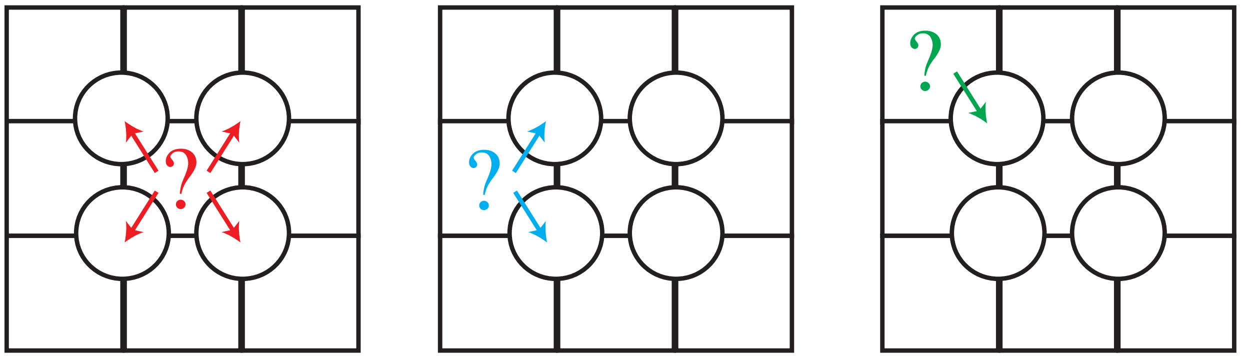
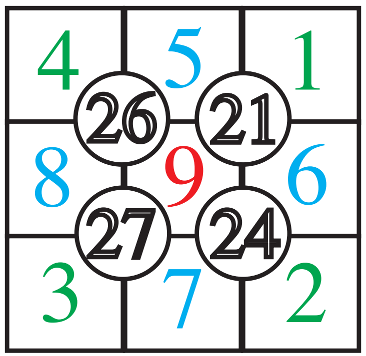
Problem 6



Example :



$$26 + 21 + 27 + 24 = 98$$

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Problem 7

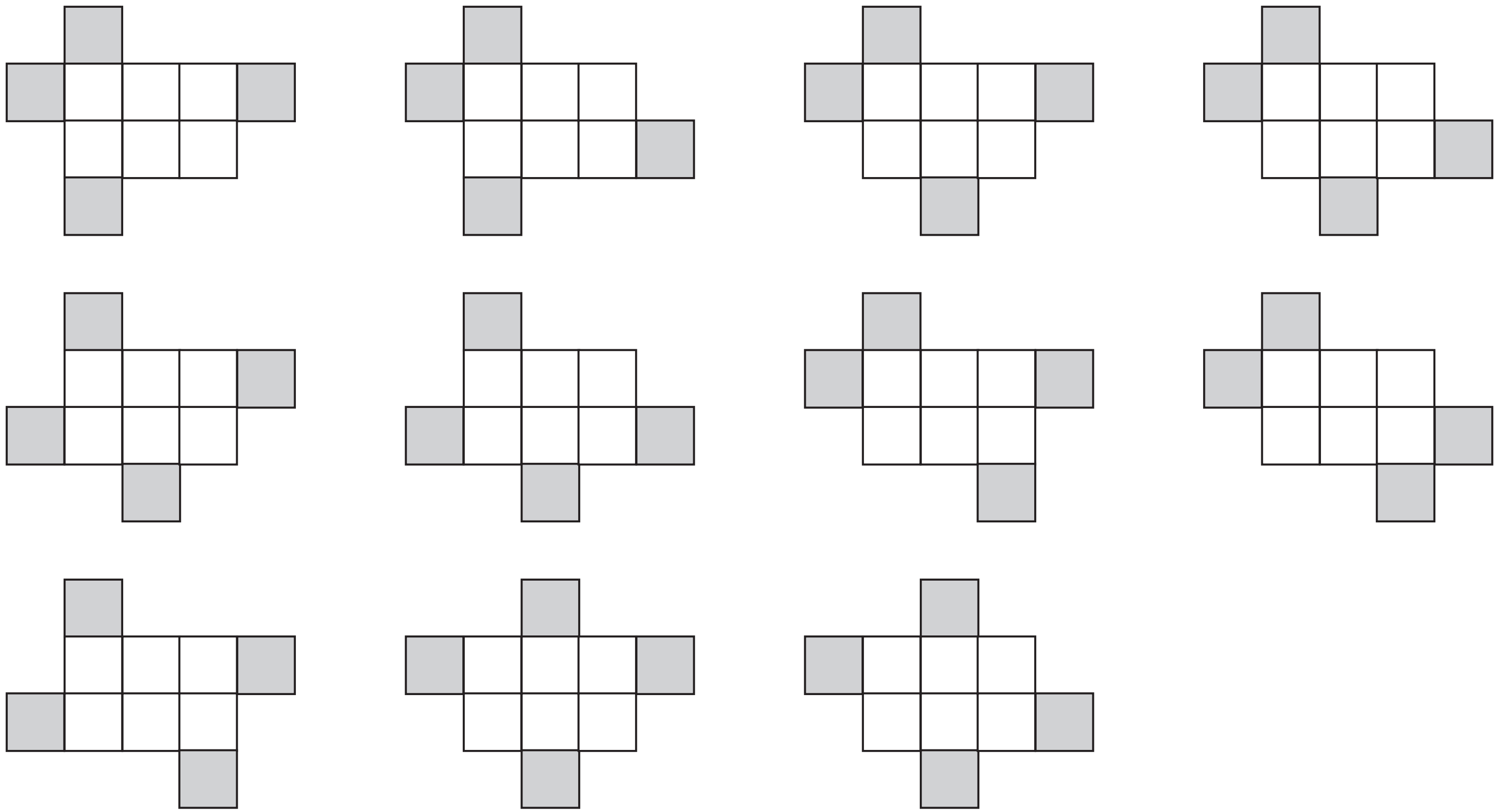
	A	B	A + B
2 min	3	4	7

350 : 7 = 50 ;

50 x 2 = 100 ;

100 min = **1 h 40 min**.

Problem 8



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Problem 9

$\overline{ab} + \overline{ba} + \overline{bc} + \overline{cb} + \overline{ac} + \overline{ca} = 20a + 20b + 20c + 2a + 2b + 2c$

$= 22a + 22b + 22c = 22(a + b + c)$.

$6 \leq a + b + c \leq 24$; $22 \times 24 = 528 \longrightarrow a \leq 5$; $a + b + c \leq 22$;

$22 \times 22 = 484 \longrightarrow a \leq 4$; $a \leq 5$; $6 \leq a + b + c \leq 22$.

a+b+c	x 22
6	**132**
7	154
8	176
9	198
10	220
11	242

a+b+c	x 22
12	**264**
13	286
14	308
15	330
16	352
17	374

a+b+c	x 22
18	**396**
19	418
20	440
21	462
22	484

Problem 10

	Σ x 2,5	digits	Σ digits
1 → 9	22,5	9	9
10 → 99	247,5	180	189
100 → 200	500	303	492
+ 1	+ 2,5	+ 3	+ 3
………	………	………..	……….
216	**540**	+ 48	**540**

Problem 11

n	*n*/50	
111 → 199	< 4	111/1 = 111 ; ☺ 112/2 = 56 ☺
211 → 299	< 6	212/4 = 53 ☺
311 → 399	< 8	312/6 = 52 ☺
411 → 499	< 10	412/8 = 51,5 ☹
511 → 599	< 12	impossible
611 → 699	< 14	612/12 = 51 ☺
711 → 799	< 16	711/7 = 101,5… 712/14 = 50,8.. ☹
811 → 899	< 18	812/16 = 50,75 ☹
911 → 999	< 20	911/9 912/18 921/18 ☹

111 ; 112 ; 212 ; 312 ; 612

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Problem 12

$$
\begin{array}{r}
\text{POŁAND} \\
\times \qquad\qquad 3 \\
\hline
= \text{WROCŁAW}
\end{array}
$$

P≥3 ; W = 1 or 2 ; D ≠ 5 ; C = 1 or 2 ; O = 5 ;

3P + 1 = $\overline{WR}$.

C	W	D	A	N	R
1	2	4	3	impossible	
2	1	7	6	8	3

P = 4 ; R = 3.

WROCŁAW = **1352061**.

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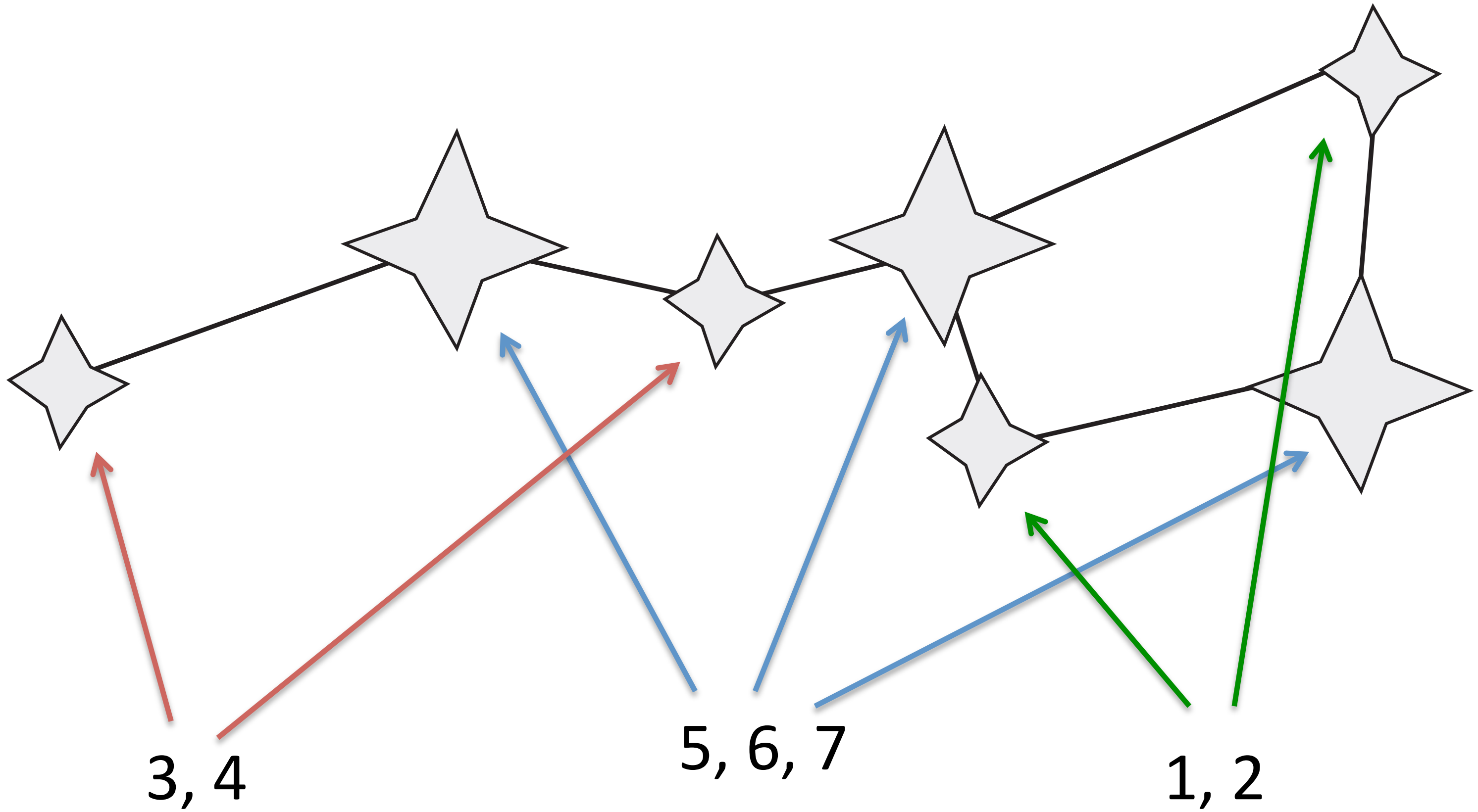
Problem 13



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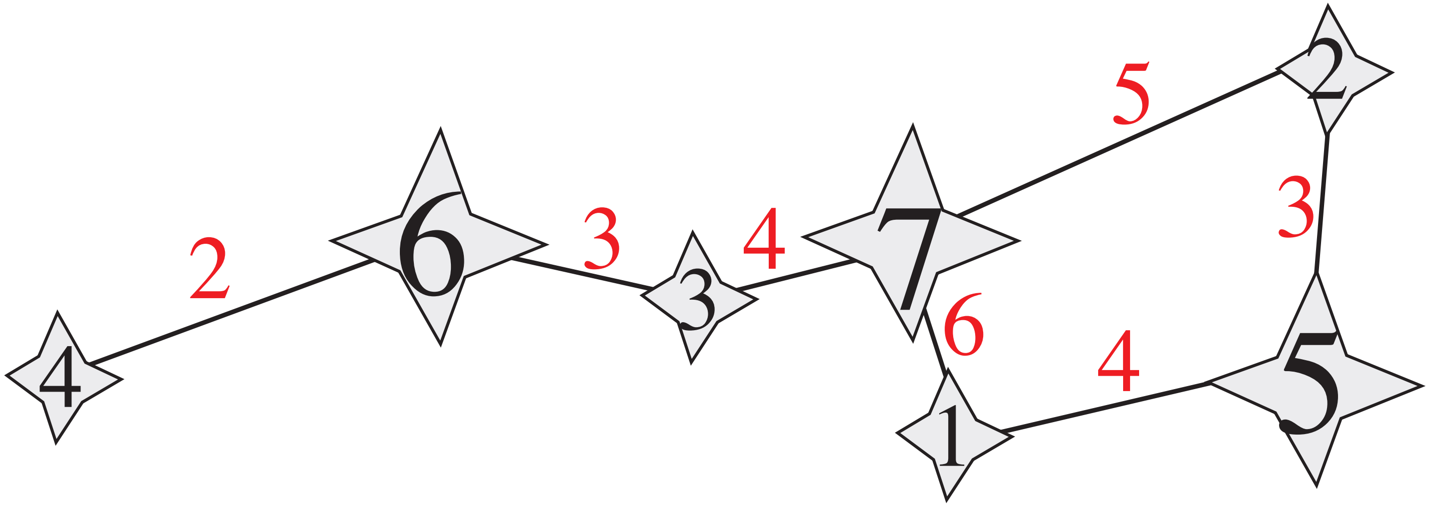
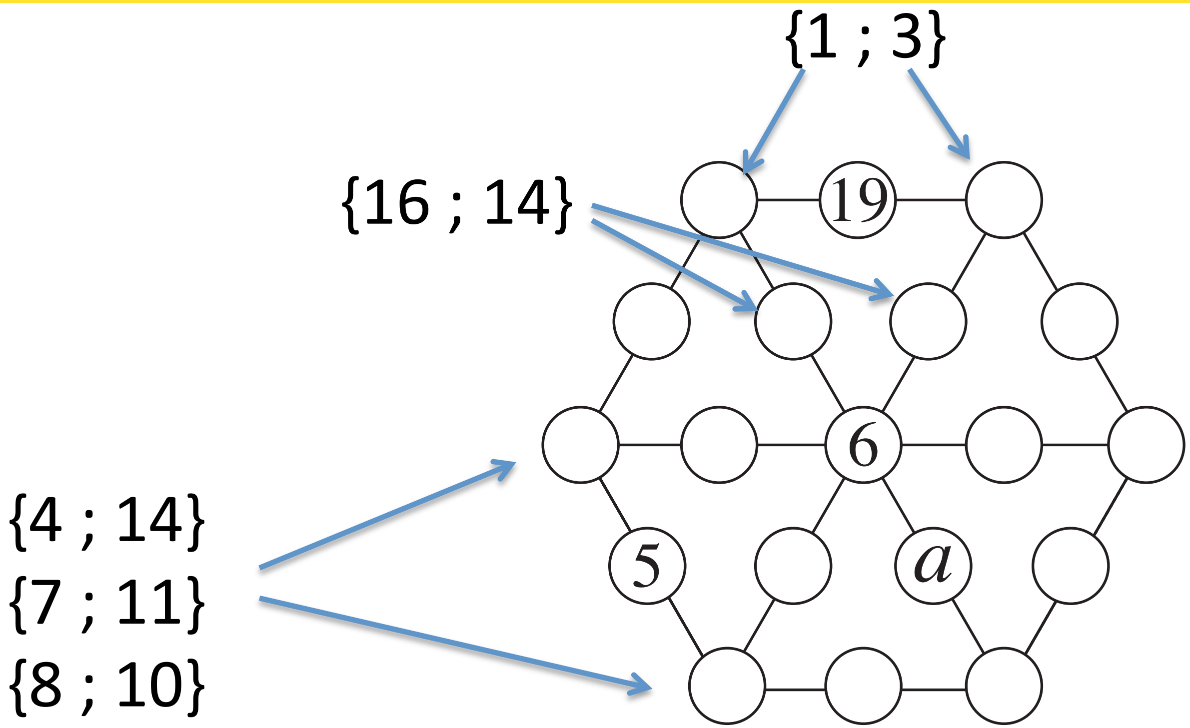
Problem 13



2 + 3 + 4 + 5 + 3 + 4 + 6 = **27**

Problem 14



{4 ; 14} impossible : 23 – (6 + 14) = 3 ;

{7 ; 11} impossible : 23 – (6 + 11) = 6.

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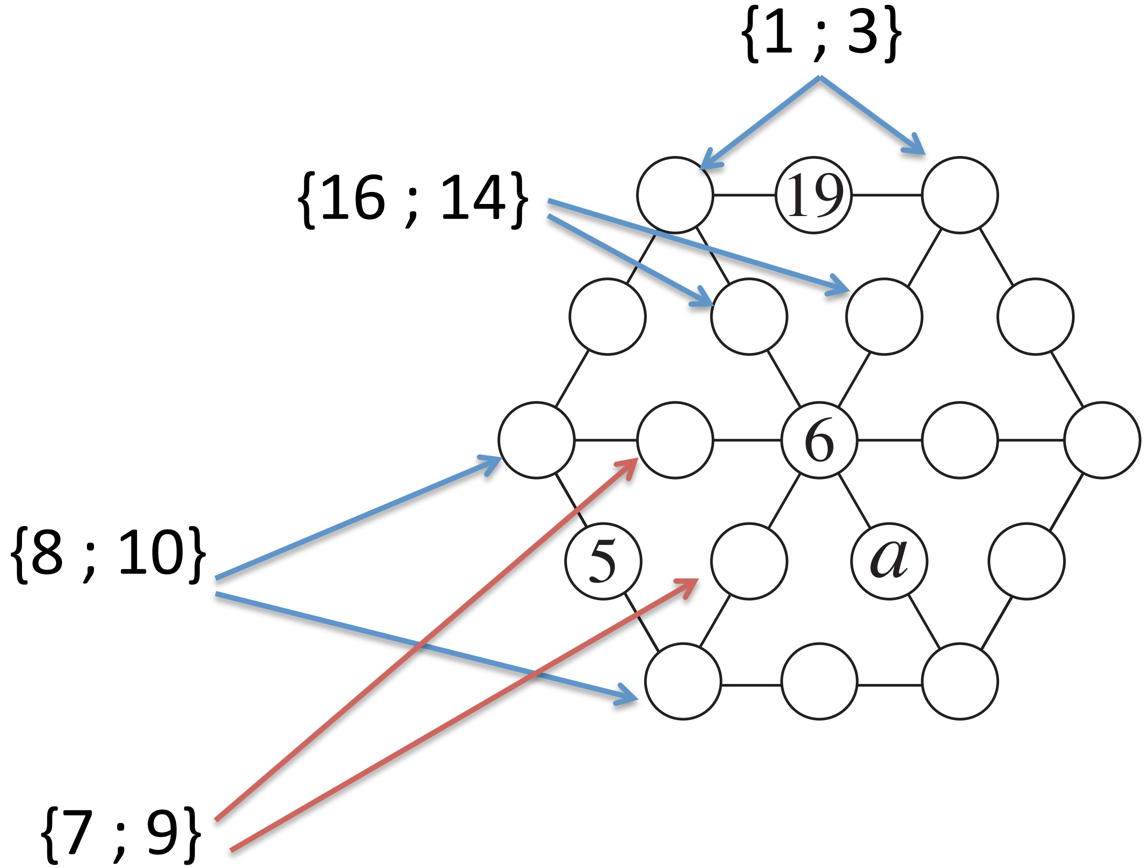
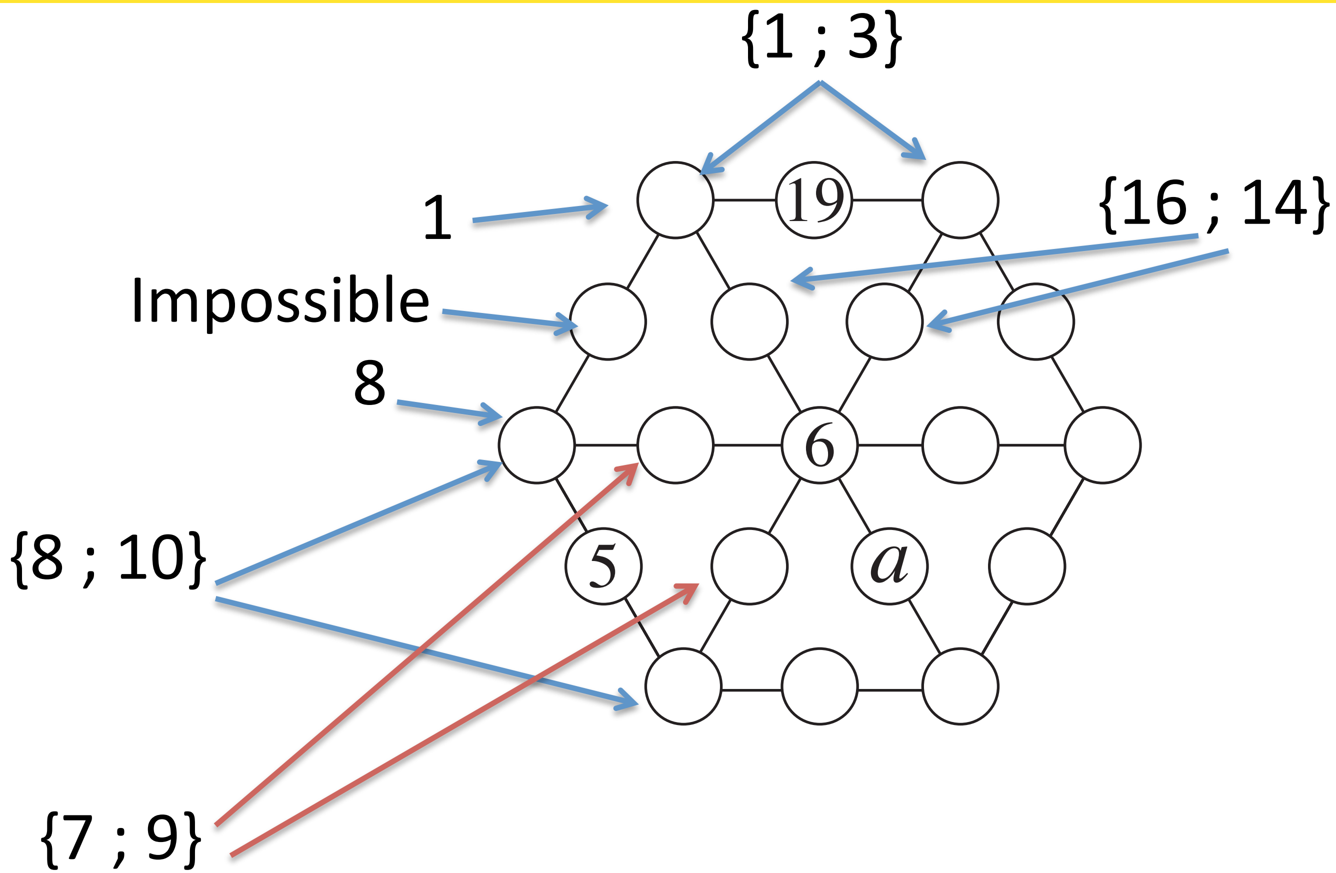
Problem 14



Problem 14



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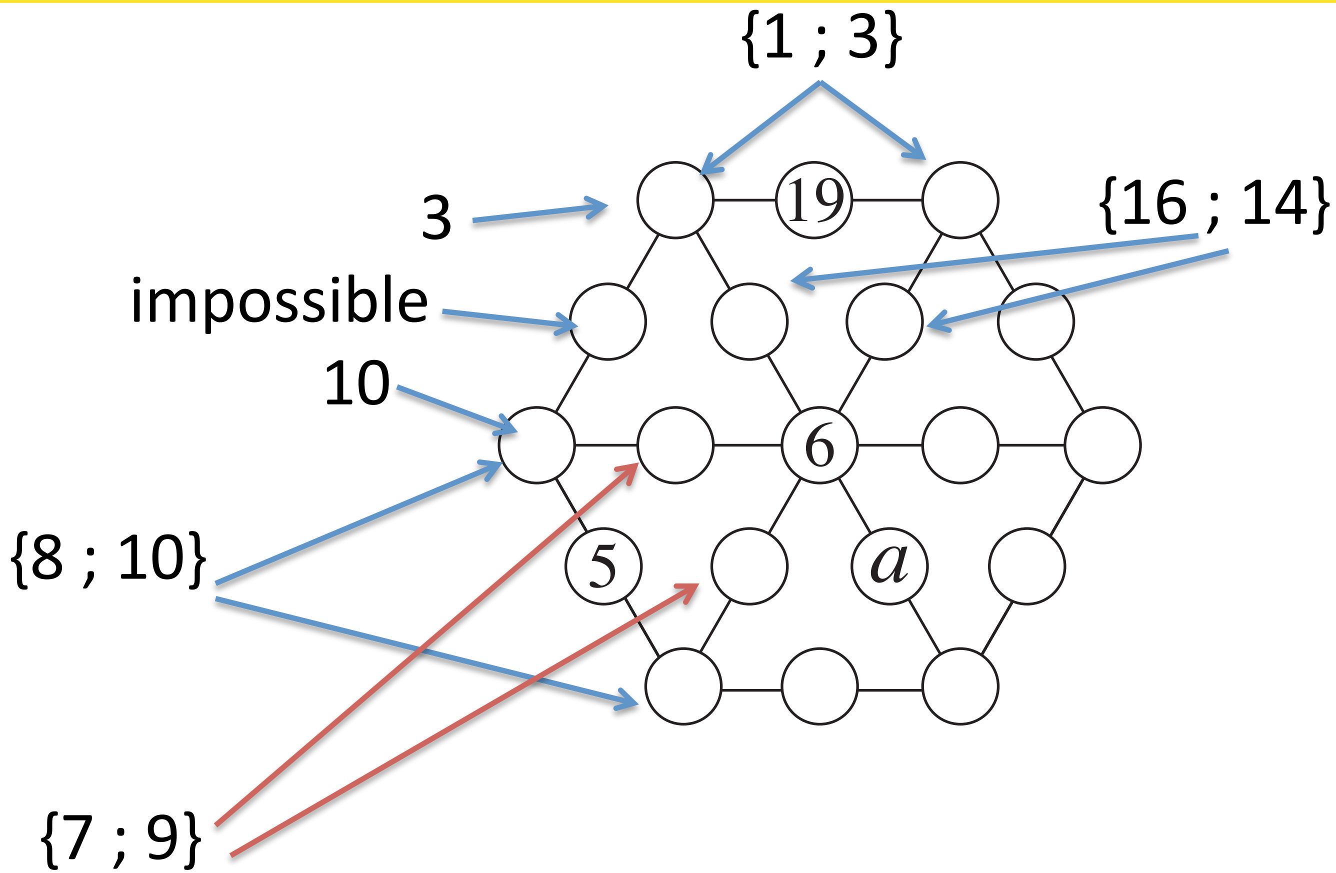

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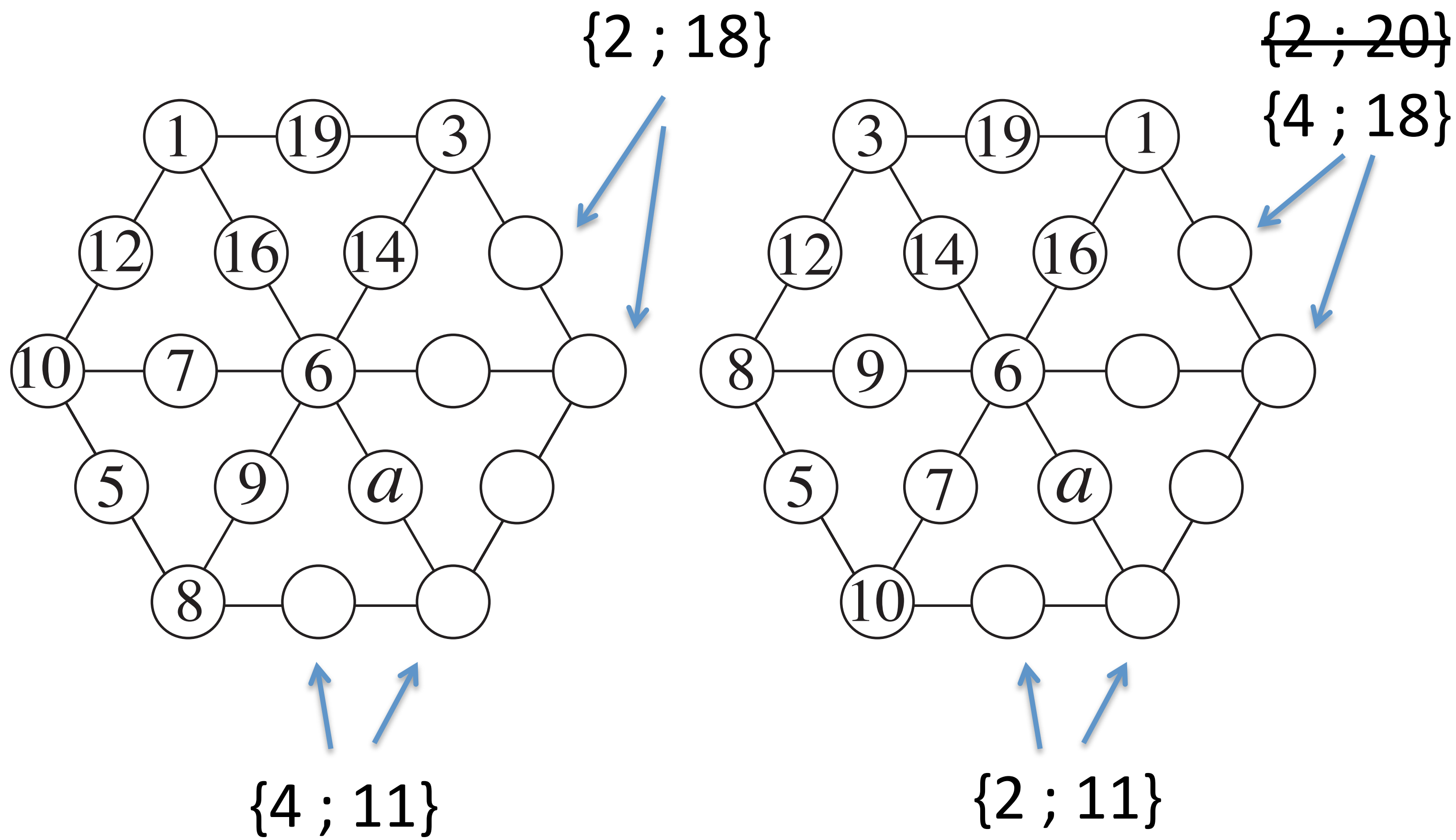
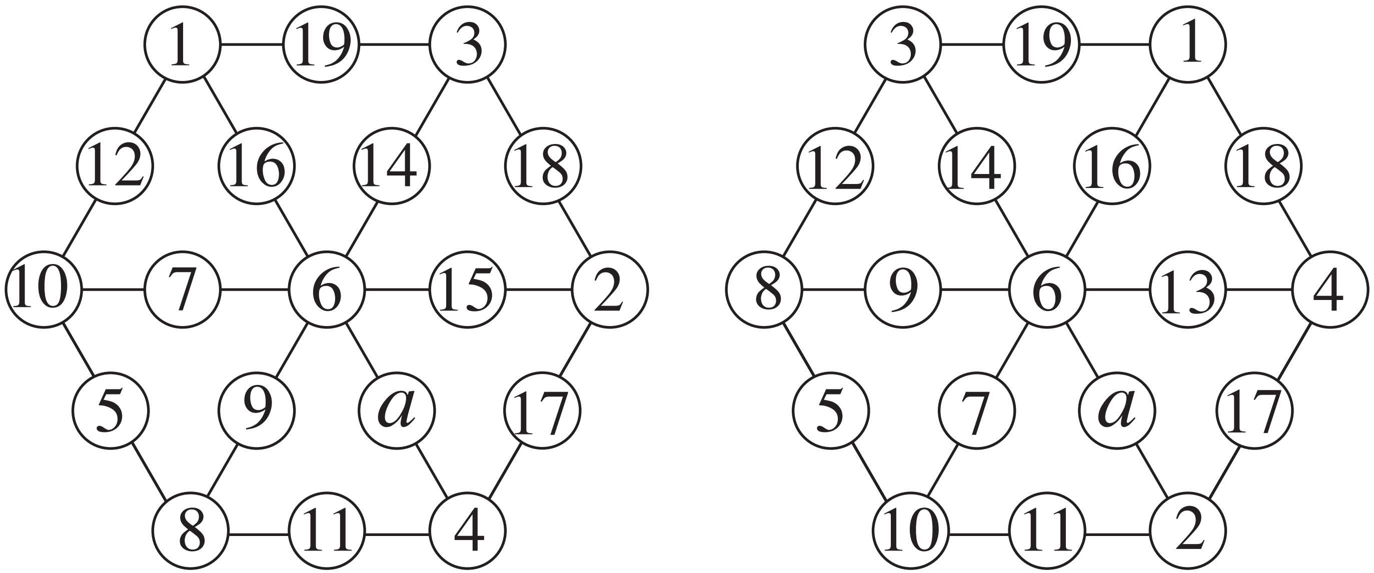


Problem 14


1
19
3
12
16
14
18
10
7
6
15
2
5
9
a
17
8
11
4
3
19
1
12
14
16
18
8
9
6
13
4
5
7
a
17
10
11
2

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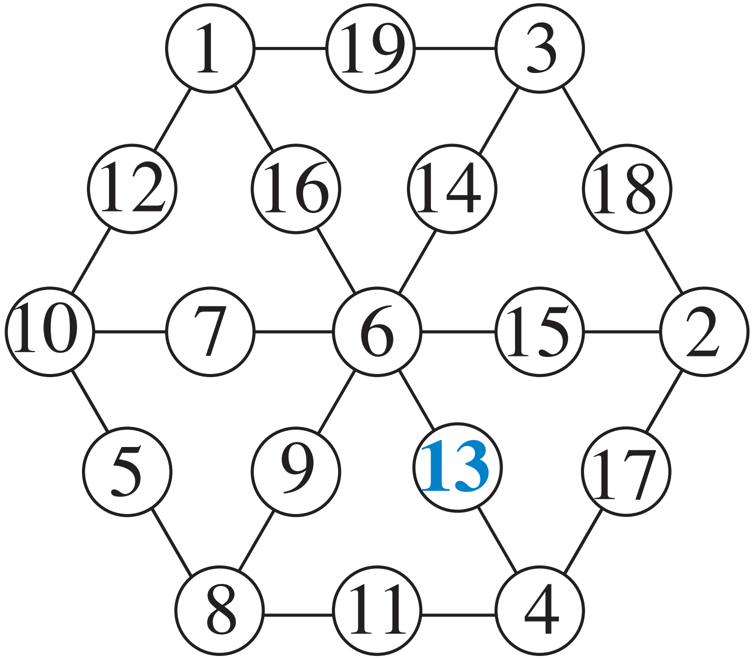
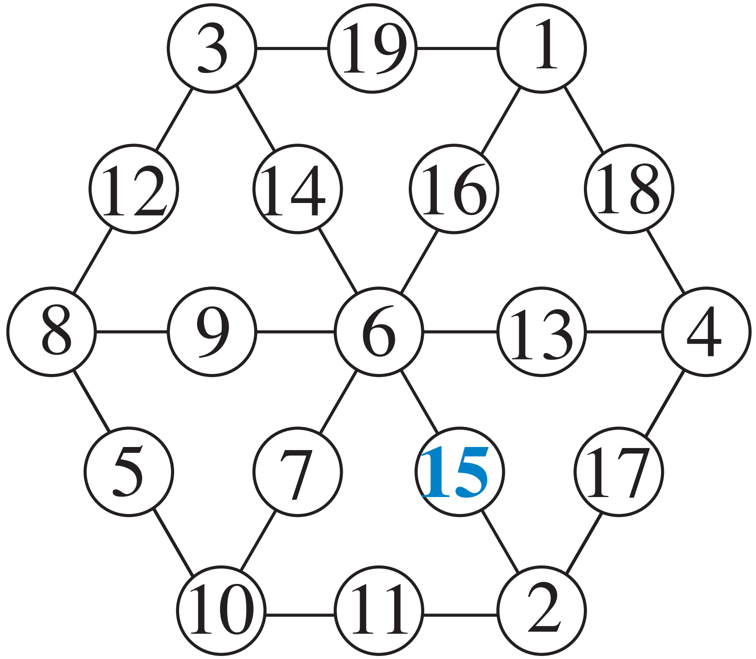
Problem 14





Problem 15

3 → 7	**$2^1 = 2$**
33 → 77	$2^2 = 4$
333 → 777	$2^3 = 8$
3333 → 7777	$2^4 = 16$
33333 → 77777	$2^5 = 32$
333333 → 777777	$2^6 = 64$
3333333 → 7777777	$2^7 = 128$
33333333 → 77777777	$2^8 = 256$
333333333 → 777777777	$2^9 = 512$
3333333333 → 7777777777	$2^{10} = 1024$

$$2 + 4 + 8 + 16 + 32 + 64 + 128 + 216 + 512 + 1024 = 2047$$

Problem 15

7777733333	2015
7777733337	2016
7777733373	2017
7777733377	2018
7777733733	2019
7777733737	2020
7777733773	2021
7777733777	2022
7777737333	2023

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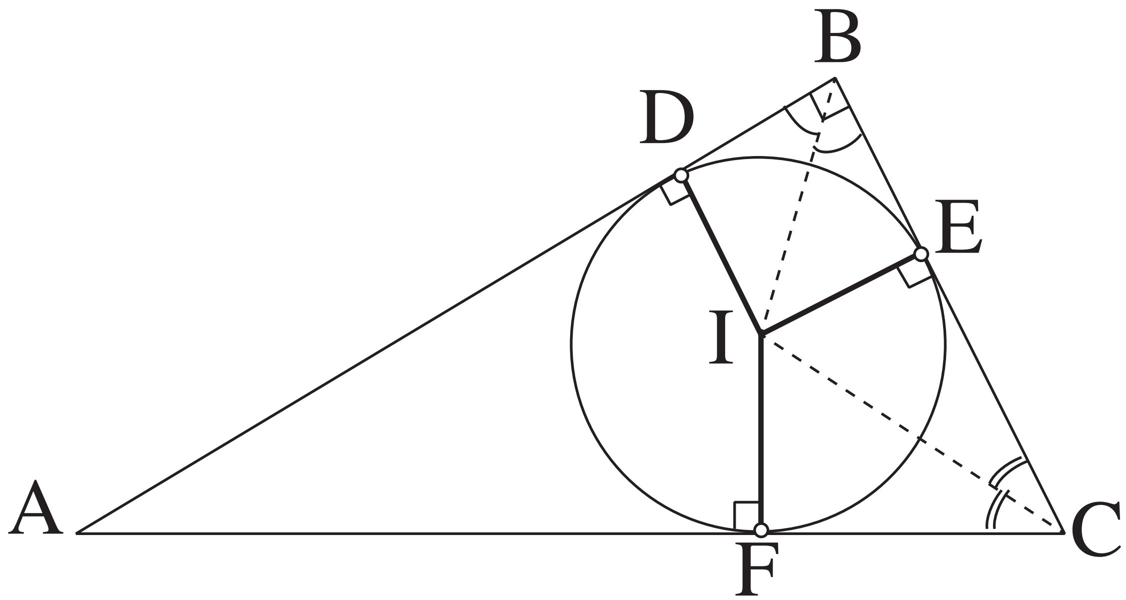
Problem 16

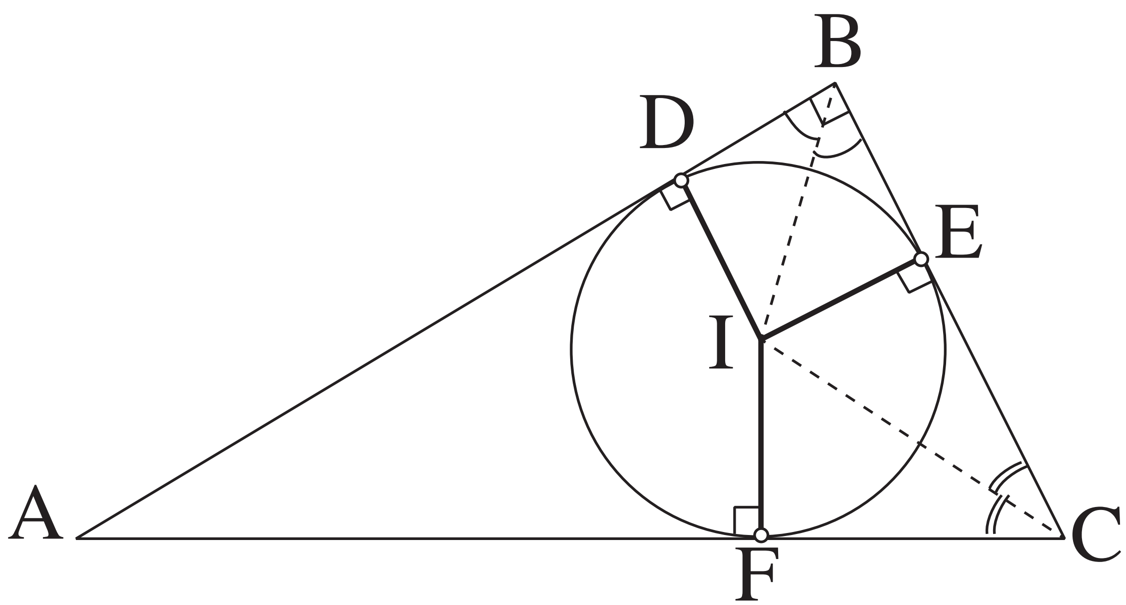


ID = IE = IF = √676 = 26 m.

area IECF = IE x EC = 1014 m^2 → CF = EC = 1014/26 = 39 m ;

BC = 26 + 39 = 65 m.

Problem 16



Pythagoras → $AC^2 = AB^2 + BC^2$; AD = AF = x ;

$(x + 39)^2 = (x + 26)^2 + 65^2$ → 26x = 3380 ; x = 130 ;

area (ADIF) = AF x IF = 130 x 26 = **3380 m²**.

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Problem 17

$$F(n) \equiv 0 \pmod{2} \Leftrightarrow n \equiv 0 \pmod{3}$$
$$F(n) \equiv 0 \pmod{5} \Leftrightarrow n \equiv 0 \pmod{5}$$
$$F(n) \equiv 0 \pmod{10} \Leftrightarrow n \equiv 0 \pmod{15}$$
$$F(n) \equiv 0 \pmod{25} \Leftrightarrow n \equiv 0 \pmod{25}$$
$$F(n) \equiv 0 \pmod{50} \Leftrightarrow n \equiv 0 \pmod{75}$$

n	25	50	75	100	125	**150**	175	200	225	250	275	300	325
F(n) Mod(100)	25	25	50	75	25	**0**	25	25	50	75	25	0	25

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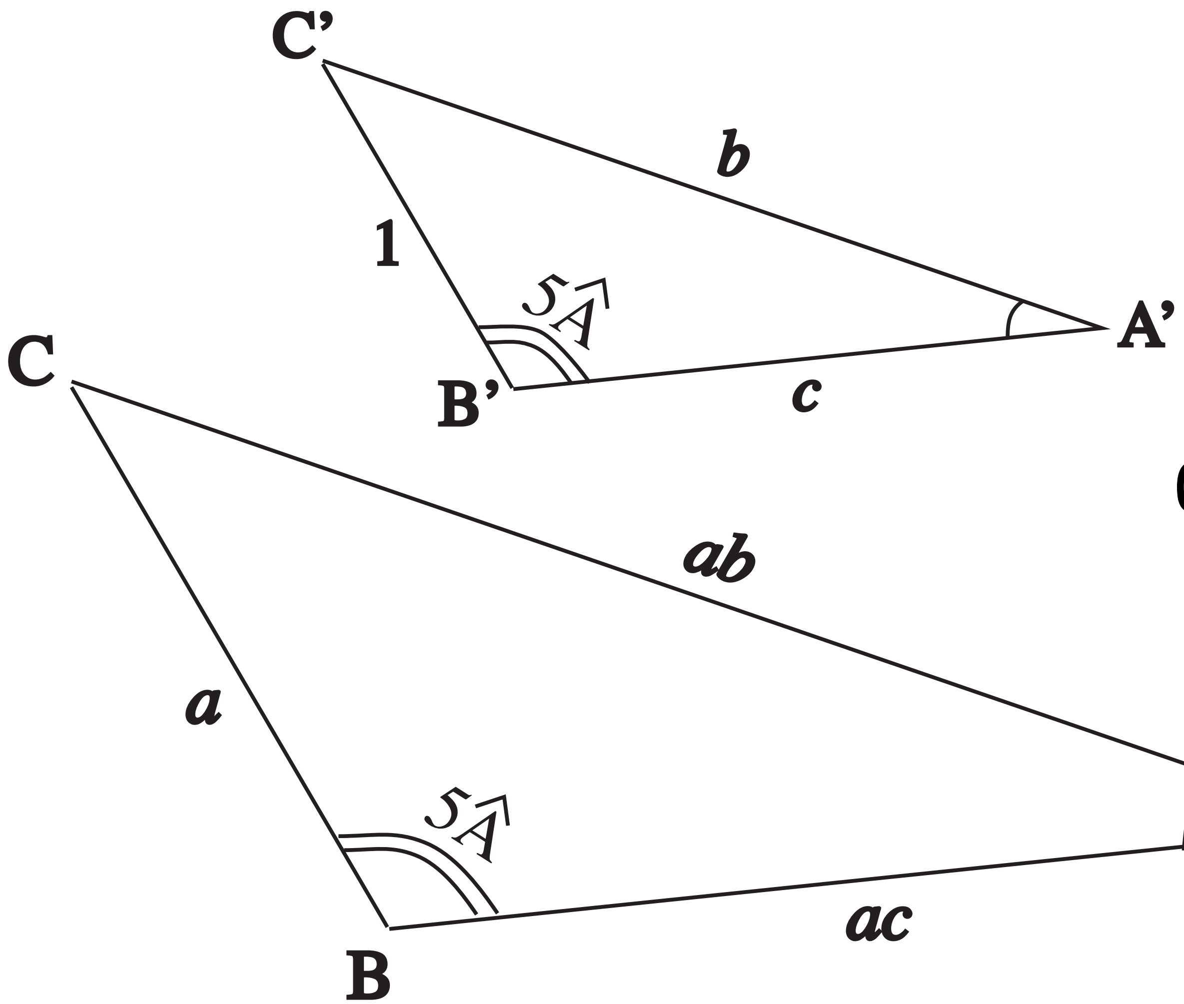
Problem 18



$$0 < \hat{A} + \hat{B} = 6\,\hat{A} < \pi$$

$$\hat{C} = \pi - 6\,\hat{A}$$

$$\hat{A} < \pi/6$$

$$\cos A > \sqrt{3}/2$$

Problem 18

$$b = \frac{\sin(5A)}{\sin A} \qquad c = \frac{\sin(6A)}{\sin A}$$

$$b = \frac{\sin(5A)}{\sin A} = (2\cos A)^4 - 3(2\cos A)^2 + 1$$

$$c = \frac{\sin(6A)}{\sin A} = (2\cos A)^5 - 4(2\cos A)^3 + 6\cos A$$

$\sqrt{3} < 2\cos A = p/q < 2$

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Problem 18

$$b = \frac{\sin(5A)}{\sin A} = (2\cos A)^4 - 3(2\cos A)^2 + 1$$

$$c = \frac{\sin(6A)}{\sin A} = (2\cos A)^5 - 4(2\cos A)^3 + 6\cos A$$

$\sqrt{3} < 2\cos A = p/q < 2$

$P = 7$; $q = 4$; $7/4 > \sqrt{3}$.

	b	***c***
1	$(7/4)^4 - 3(7/4)^2 + 1 = 305/256$	$(7/4)^5 - 4(7/4)^3 + 21/4 = 231/1024$

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Problem 18

1	*b*	*c*
1	$(7/4)^4 - 3(7/4)^2 + 1 = 305/256$	$(7/4)^5 - 4(7/4)^3 + 21/4 = 231/1024$

BC = *a*	AB = *ac*	AC = *ab*
1024	305 x 4 = 1220	231

Bertrand – Camille = 231 m or 462 m ;

Aliénor – Camille = **1024 or 1220 or 2048 m or 2440 m.**