EN

37th International Championship for Mathematical and Logical Games, 25.08.2023

Information and results at http://gmil.pwr.edu.pl/

START for ALL PARTICIPANTS

1. FIVE NUMBERS (coefficient 1)

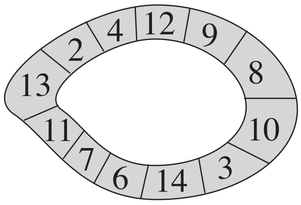
13 ; 19 ; 44 ; 114 ; 15

We add the same number, smaller than 23, to each of these five numbers. The five results all contain at least one digit 2.

What number was added?

2. CUT THE BAND (coefficient 2)

This band can be cut into three pieces, the sum of the numbers written on each of these three pieces being the same.

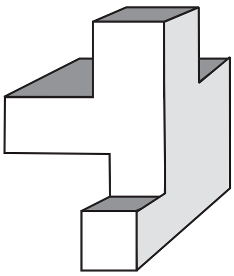


Which numbers will be on the same piece as 13? *Write these numbers in order from smallest to largest.*

3. STRANGE BUILDING (coefficient 3)

All the faces of this building are horizontal or vertical.

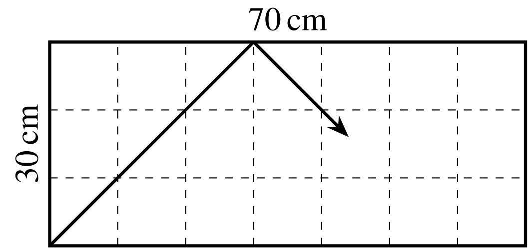
How many faces does it have, at least, counting the face in contact with the ground?



4. INCREASING NUMBERS (coefficient 4)

How many five-digit numbers are there where each digit, except the rightmost digit, is greater than the sum of the digits (or the digit if there is only one) to its right?

5. PAULA'S POOL TABLE (coefficient 5)



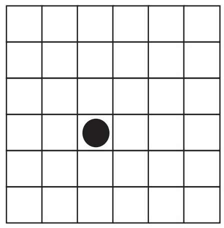
Paula has received a rectangular pool table measuring 70 cm by 30 cm. She plays a ball from a corner of the table following a diagonal of a small square. When the ball hits a cushion, it always rebounds following a diagonal of a small square.

How many rebounds will the ball make before entering a corner pocket?

END for CE PARTICIPANTS

6. COUNTING COUNTERS ON A GRID (coefficient 6)

On this grid Matilda has placed a counter. She wants to add counters according to the following rules:



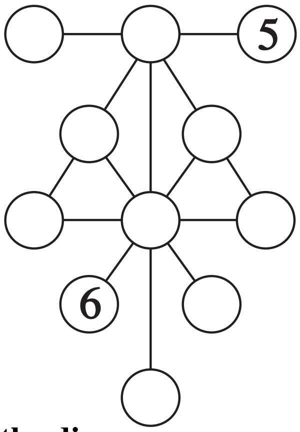
– each horizontal or vertical row must never contain more than two counters;

– squares that touch a square containing a counter, even diagonally, must be empty.

How many counters can she place at most, including the counter already placed?

7. DIAGRAM OF THE YEAR (coefficient 7)

The discs of this diagram contain all the whole numbers from 2 to 12 (the numbers 5 and 6 are already placed). The sum of three numbers located in discs placed on the same line segment is always equal to 23.



Fill in the empty discs in the diagram.

8. CARPOOLING (coefficient 8)

A city would like to encourage carpooling and has decided to count the number of people who pass by car on a certain stretch of road. Of the 2550 cars counted, 1 in 25 contained 5 people, 1 in 10 contained 4, 1 in 17 contained 3 and 1 in 6 contained 2; all the others had only 1 person on board.

How many people were counted in the 2550 cars?

END for CM PARTICIPANTS

Problems 9 to 18: beware! For a problem to be completely solved, you must give both the number of solutions, AND give the solution if there is only one, or give any two correct solutions if there are more than one. For all problems that may have more than one solution, there is space for two answers on the answer sheet (but there may still be just one solution).

9. FRIENDLY NUMBERS (coefficient 9)

Two friendly numbers are numbers such that each is divisible by the sum of the digits of the other. A and B are friendly numbers, both between 100 and 150.

What is the number $A + B$, knowing that B is divisible by 23?

10. INTERLEAVING (coefficient 10)

A, B, X and Y are four different digits, A and B being different from 0.

$$AB \times BA = AXYB$$

By multiplying the two-digit number AB by its "reverse", we obtain a four-digit number in which the digits X and Y are inserted between A and B.

What is the number $AXYB$?

11. MORE MAGICAL SQUARE (coefficient 11)

Matthew filled the nine boxes of a 3×3 square with the integers from 1 to 9 so that the six sums obtained for the rows and columns are all different. The largest of its six sums is 24.

2	1	6	→ 9
3	4	5	→ 12
9	8	7	→ 24
↓ 14	↓ 13	↓ 18	

Matilda did the same thing, but the largest of her six sums is smaller than 24.

What is this larger sum, at least?

END for C1 PARTICIPANTS

12. CRYPTARITHM (coefficient 12)

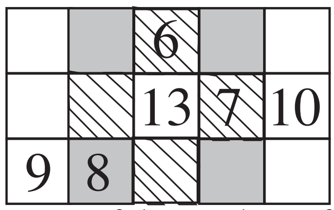
In this cryptarithm the same letter always replaces the same digit and the same digit is always replaced by the same letter. The first digit of a multi-digit number cannot be a 0.

$$\frac{\text{WROC}}{\text{ŁAW}} = \frac{4}{3}$$

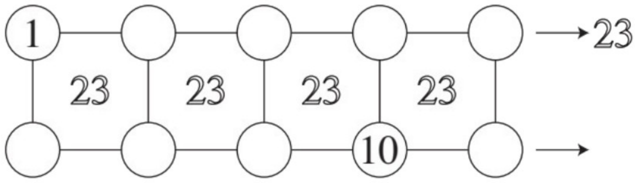
What is the number WROCŁAW?

13. MAGIC RECTANGLE (coefficient 13)

Complete this rectangle which must contain 15 consecutive integers. The central value, 13, must be the average of the numbers of each column, of each line, of the 4 corners, of the 4 middles of the four sides, and also the average of the numbers of the 4 shaded boxes and that of the numbers of the four hatched boxes.



14. FROM ONE TO TEN (coefficient 14)



Complete the empty discs of this diagram with the numbers from 2 to 9 (1 and 10 are already placed) so that:

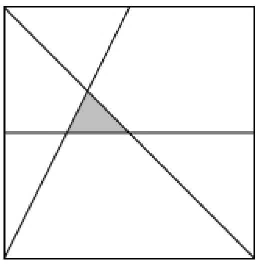
– the sum of the four numbers located at the vertices of each square is 23;

– the sum of the five numbers in the top row is 23.

END for C2 PARTICIPANTS

15. MATTHEW'S GARDEN (coefficient 15)

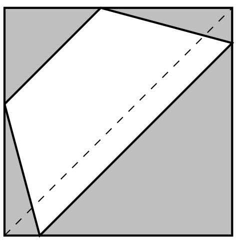
Matthew has a square garden crossed by three streams as in the drawing. The streams enter and leave the garden either at a corner or at the middle of a side. Matthew is making his vegetable garden in the little triangle between the three streams. Knowing that his vegetable garden is 10 m^2,



what is the area of Matthew's garden (in m^2)?

16. NATHALIE'S CUTOUT (coefficient 16)

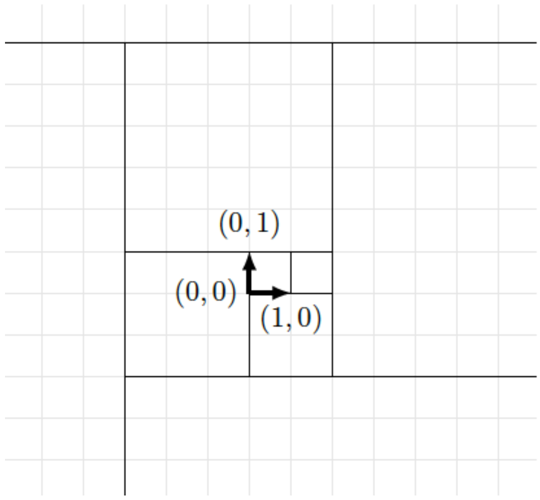
Nathalie cuts out the shaded parts of a square sheet to obtain a trapezium equal to half of a regular convex hexagon. The two bases of the trapezium are parallel to a diagonal of the square and all its vertices are placed on the sides of the square. The total area of the four rejected parts is 216 cm^2.



What is the length, in cm, of the longest side of the trapezium she cut out? *Round the answer to tenths of a cm. If necessary, take* $\sqrt{2}$ *as* 1.41 *and* $\sqrt{3}$ *as* 1.73.

END for L1, GP PARTICIPANTS

17. GROWTH (coefficient 17)

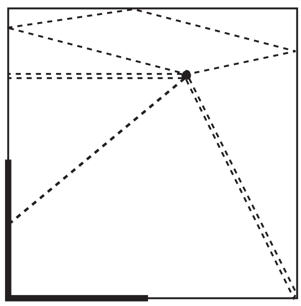


The plane is paved with squares as shown in the figure.

What is the length of the side of the square containing the point of coordinates $(2023, 2023)$?

18. CAESAR MYRIAD (coefficient 18)

Caesar Myriad is a ghost materialising in a square room whose walls are mirrors except for two mirrorless half-walls around the same corner (the bold line segments in the figure, drawn as viewed from above). If he stands as in the drawing, he sees an infinity of reflections of himself. He decides to move, without touching a wall, and notices that he only sees a finite number of reflections of himself. Looking around the room, he will see one of his reflections when he looks at one of a finite number of points on the mirrors.



What will then be the minimum number of points where he will see one of his reflections?

END for L2, HC PARTICIPANTS