

37<sup>e</sup> Championnat International  
des Jeux Mathématiques et Logiques



25 august 2023

**Solutions (RAMA)**

Problem 1

$$\begin{array}{r} 13 \\ + ? \\ \hline \end{array} \quad \begin{array}{r} 19 \\ + ? \\ \hline \end{array} \quad \begin{array}{r} 44 \\ + ? \\ \hline \end{array} \quad \begin{array}{r} 114 \\ + ? \\ \hline \end{array} \quad \begin{array}{r} 15 \\ + ? \\ \hline \end{array}$$

Problem 1

$$\begin{array}{ccccc} 13 & 19 & 44 & 114 & 15 \\ + ? & + ? & + ? & + ? & + ? \\ \hline \end{array}$$

↓

$$? > 6$$

↓

$$\begin{array}{l} ? = 8 \\ ? = 18 \end{array}$$

↓

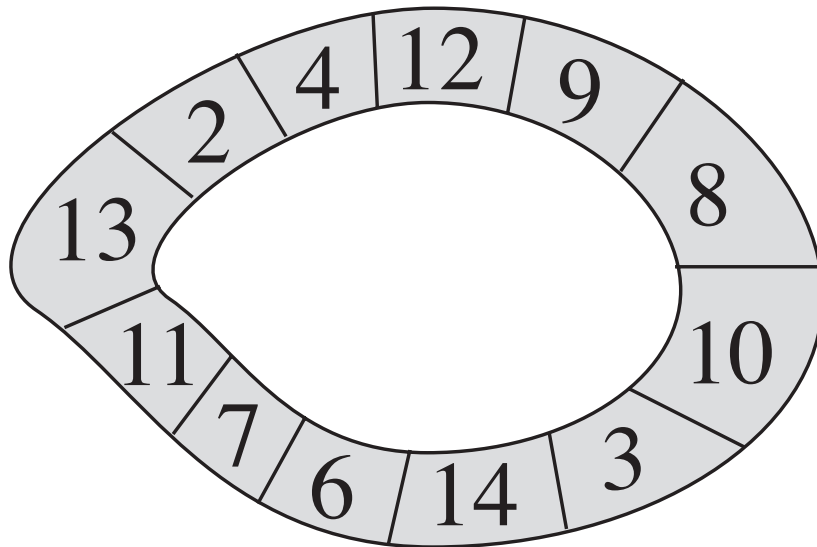
$$\begin{array}{l} 15+18 \\ = 33 \quad \text{☹} \end{array}$$

Problem 1

$$\begin{array}{r} 13 \\ + 8 \\ \hline 21 \end{array} \quad \begin{array}{r} 19 \\ + 8 \\ \hline 27 \end{array} \quad \begin{array}{r} 44 \\ + 8 \\ \hline 52 \end{array} \quad \begin{array}{r} 114 \\ + 8 \\ \hline 122 \end{array} \quad \begin{array}{r} 15 \\ + 8 \\ \hline 23 \end{array}$$



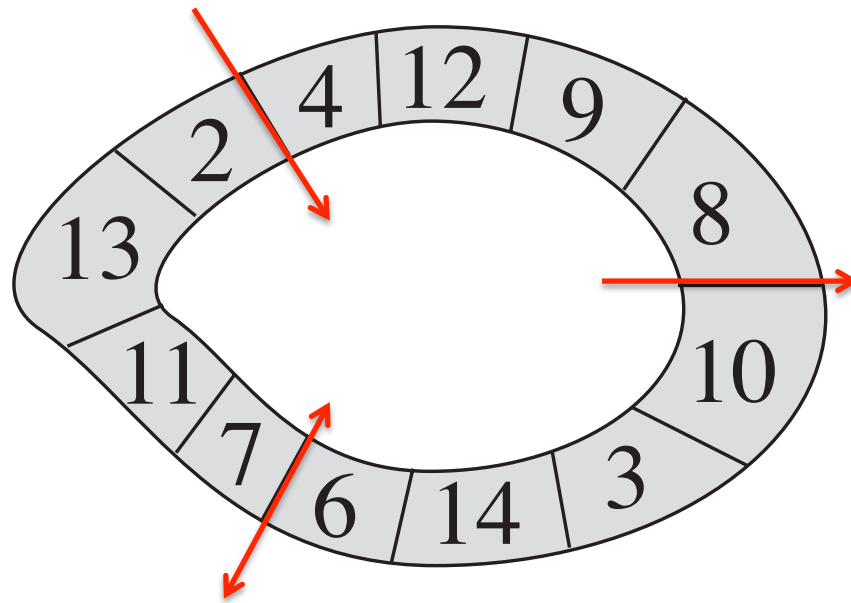
## Problem 2



$$13+2+4+12+9+8+10+3$$
$$+14+6+7+11 = 99$$
$$99 : 3 = \mathbf{33}$$

$$6 + 7 + 11 + 13 = 37 \text{ ☹️}$$
$$7 + 11 + 13 + 2 = \mathbf{33} \text{ 😊}$$
$$11 + 13 + 2 + 4 = 30 \text{ ☹️}$$
$$13 + 2 + 4 + 12 = 31 \text{ ☹️}$$

## Problem 2



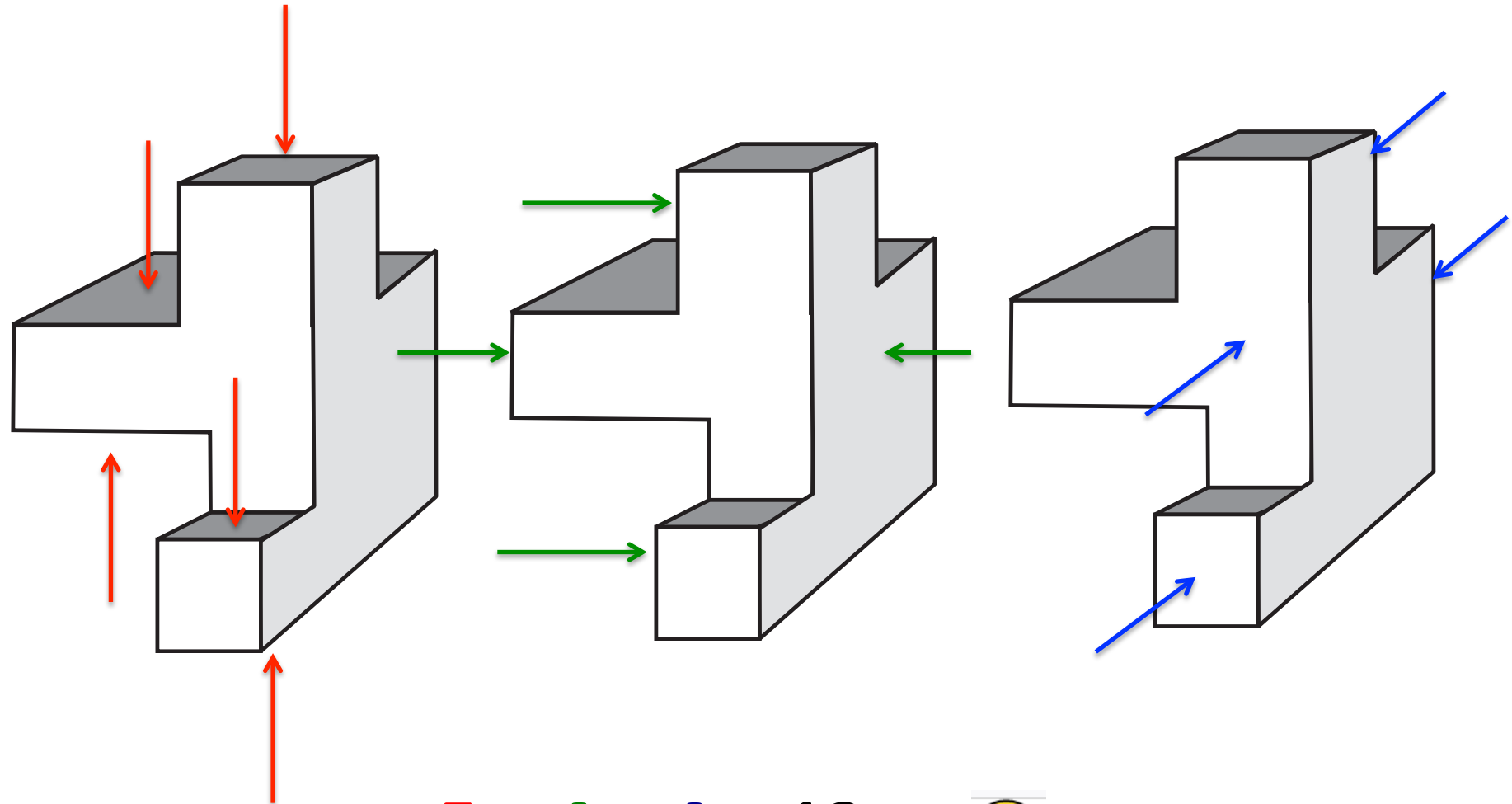
$$4 + 12 + 9 + 8 = 33$$

$$10 + 3 + 14 + 6 = 33$$

$$7 + 11 + 13 + 2 = 33$$



## Problem 3



$$5 + 4 + 4 = 13$$



## Problem 4

? 8421



3

84210

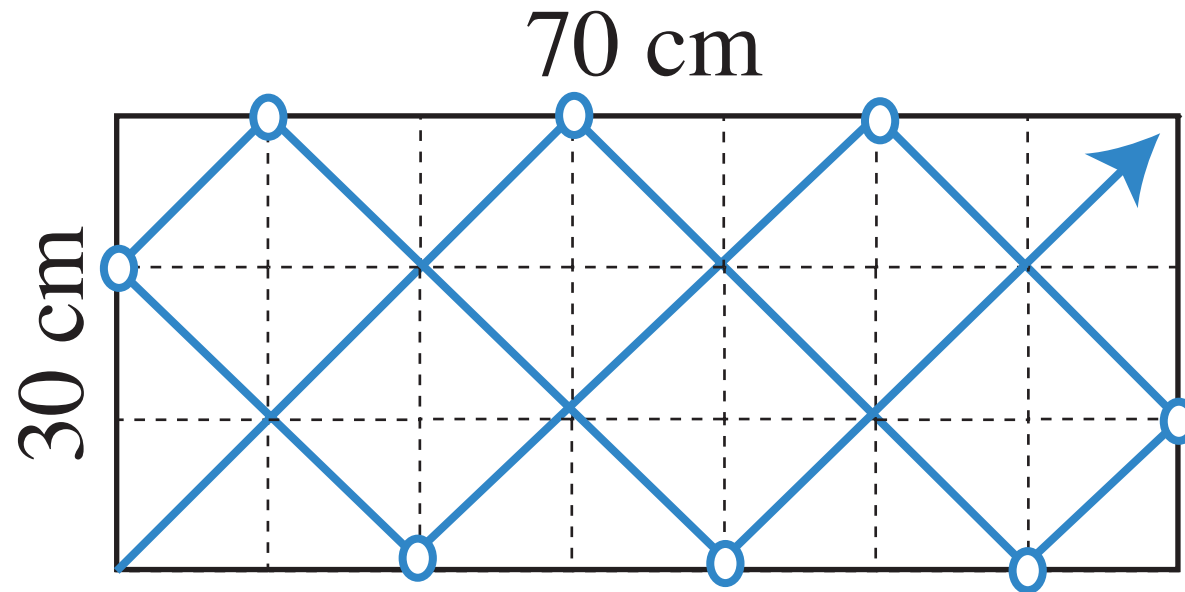
94210

95210





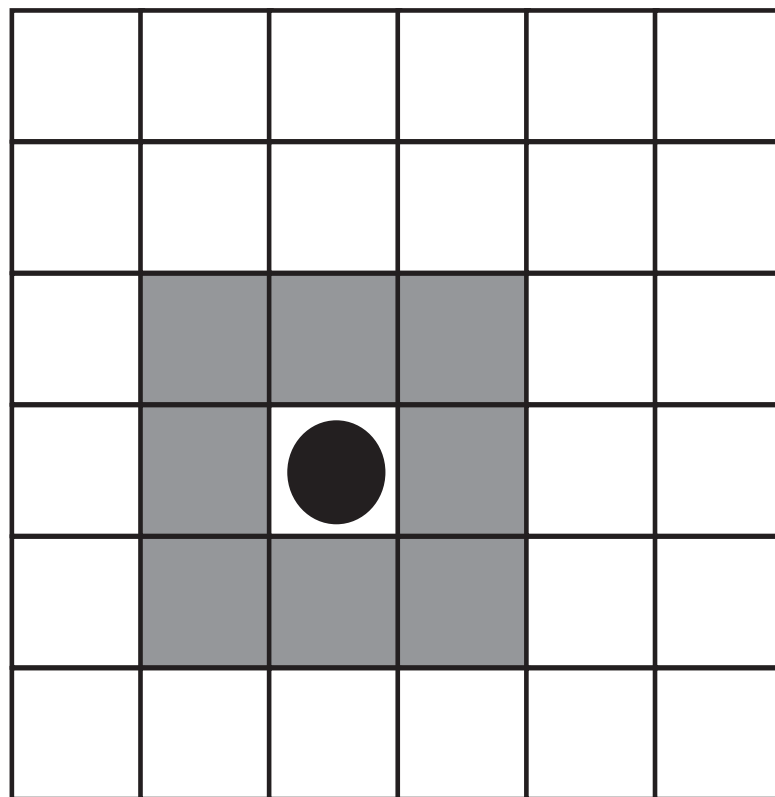
## Problem 5



8

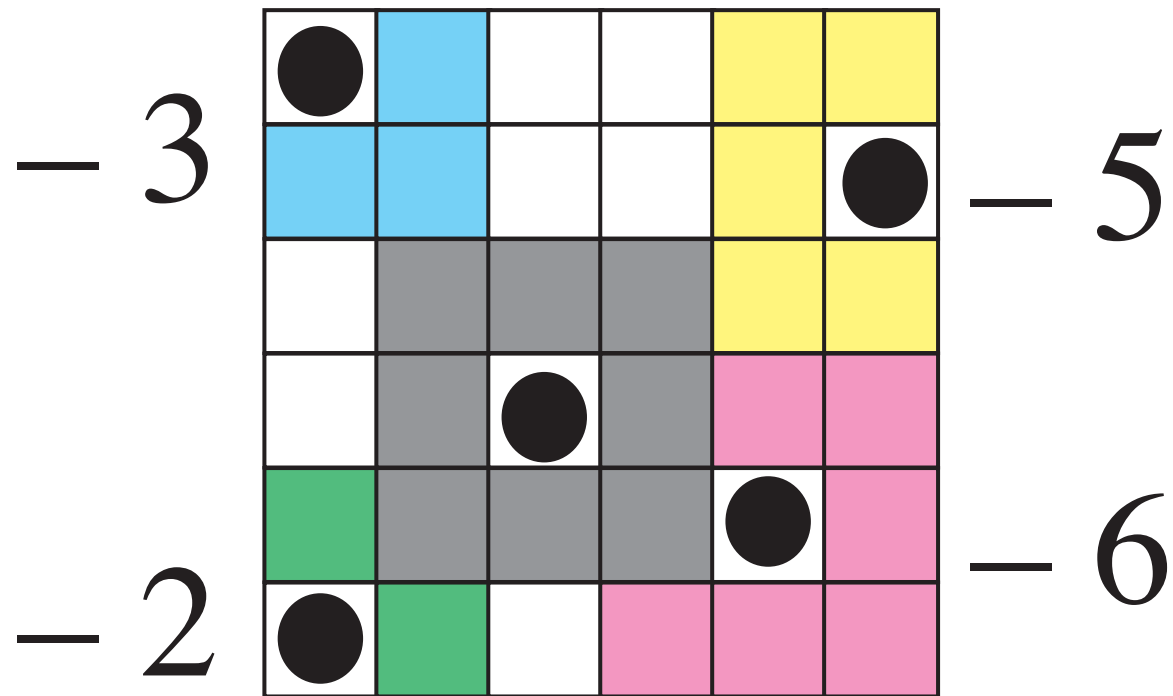


## Problem 6



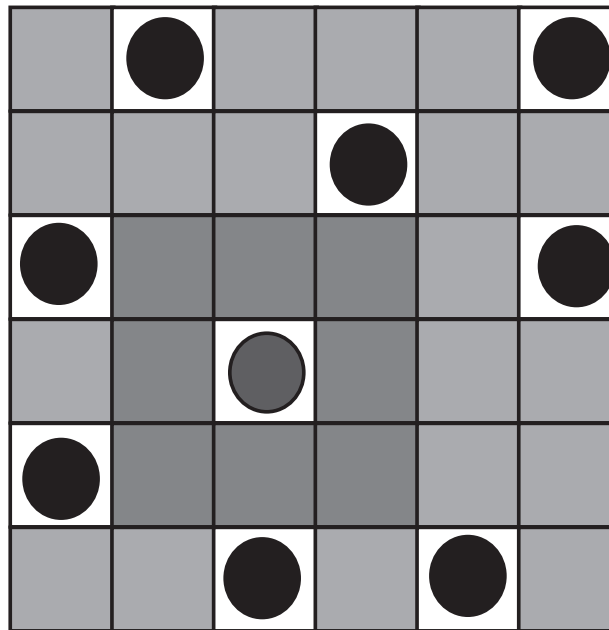
— 8

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Problem 6



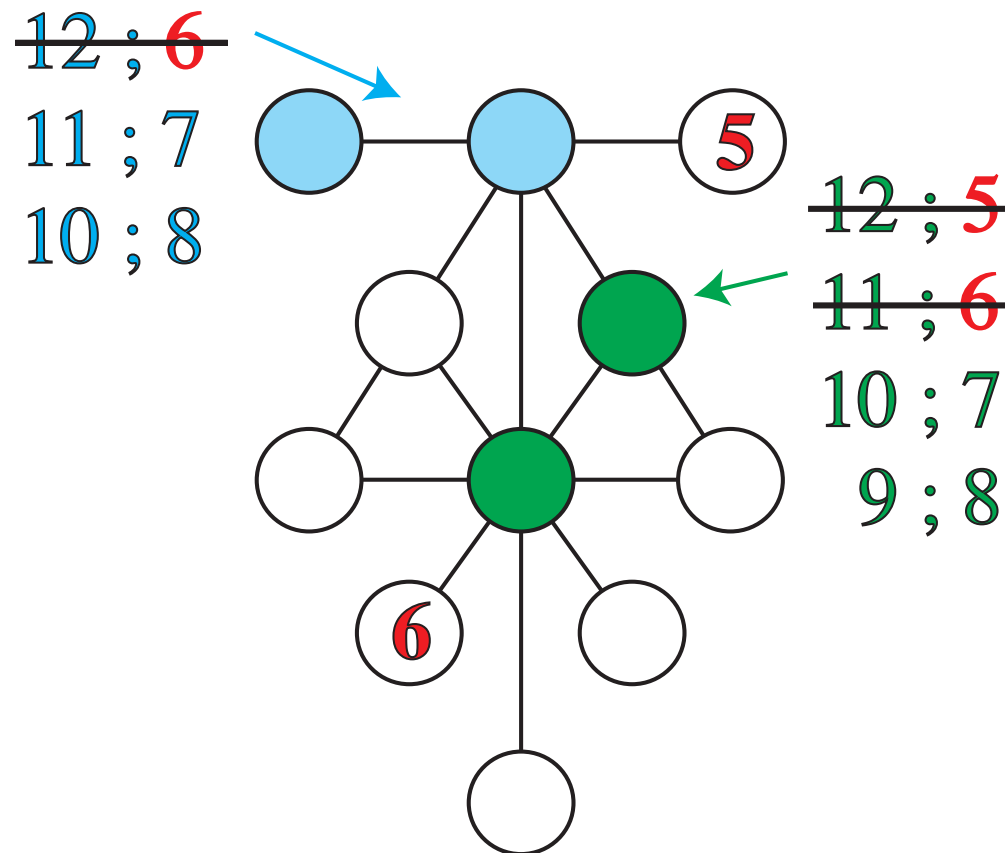
## Problem 6

### Example

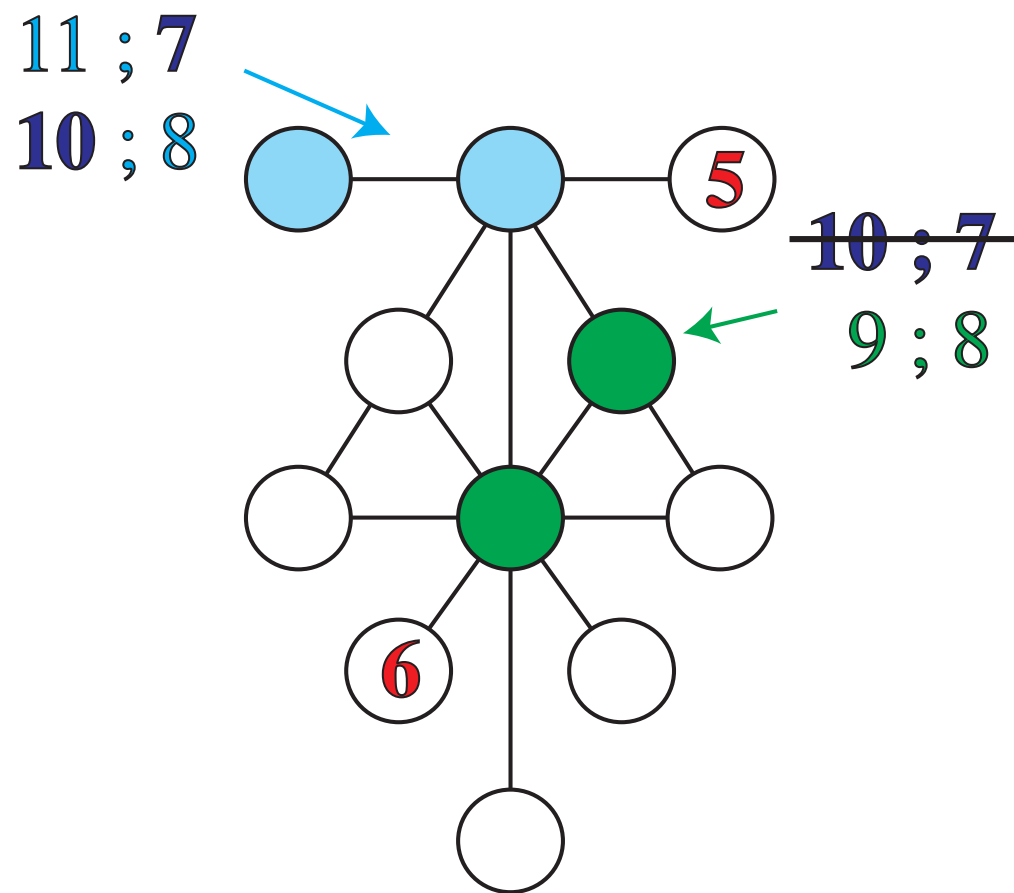


9

# Problem 7

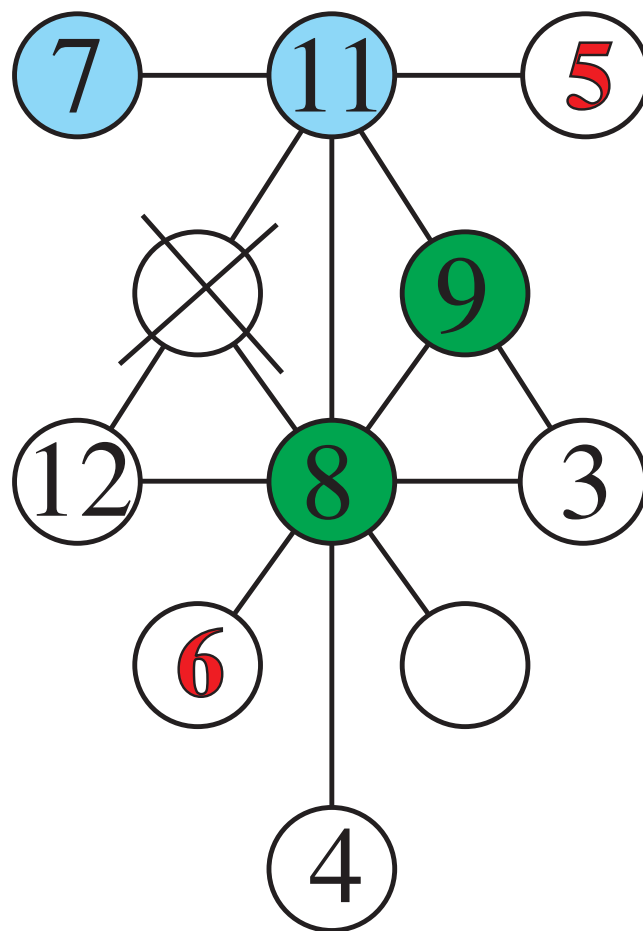


# Problem 7



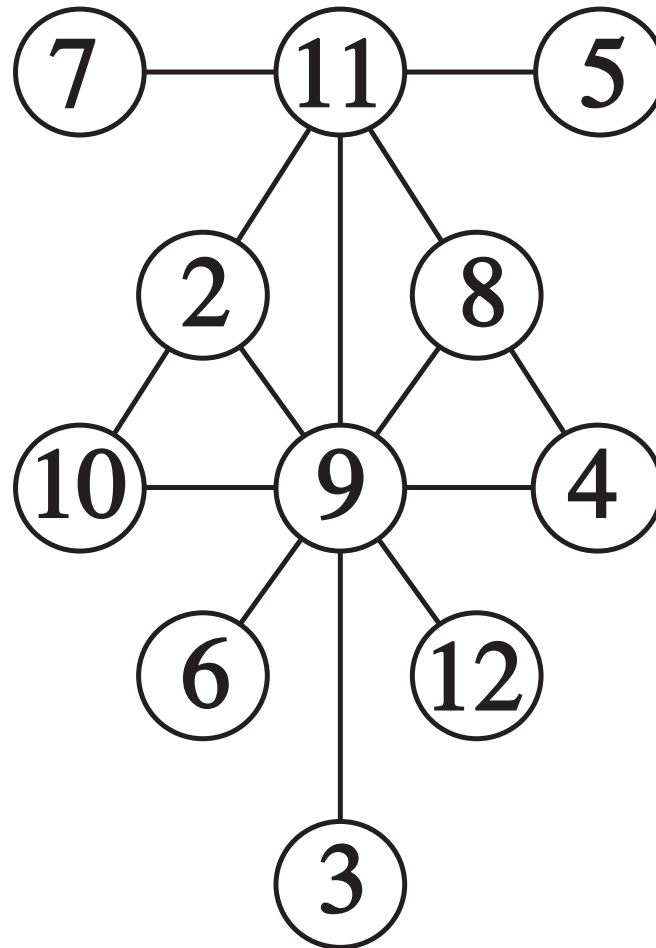


## Problem 7





## Problem 7



## Problem 8

$$2550 : 25 = 102 ;$$

$$2550 : 10 = 255 ;$$

$$2550 : 17 = 150 ;$$

$$2550 : 6 = 425.$$

$$102 + 255 + 150 + 425 = 932 ;$$

$$2550 - 932 = 1618.$$

## Problem 8

$$102 \times 5 = 510 ;$$

$$255 \times 4 = 1020 ;$$

$$150 \times 3 = 450 ;$$

$$425 \times 2 = 850.$$

$$1618 + 850 + 450 + 1020 + 510$$

$$= 4448. \quad \text{😊}$$

## Problem 9

B :  $4 \times 23 = 92 < 100$  ☹️

$5 \times 23 = 115$  ;  $1 + 1 + 5 = 7$  😊

$6 \times 23 = 138$  ;  $1 + 3 + 8 = 12$  😊

$7 \times 23 = 161 > 150$  ☹️

## Problem 9

$$B = 115 = 5 \times 23 ; 1 + 1 + 5 = 7$$

$$A : \quad 7 \times 14 < 100 ; 7 \times 22 > 150$$

$$7 \times 15 = 105 ; 1 + 0 + 5 = 6 \quad \text{☹️}$$

$$7 \times 16 = 112 ; 1 + 1 + 2 = 4 \quad \text{☹️}$$

$$7 \times 17 = 119 ; 1 + 1 + 9 = 11 \quad \text{☹️}$$

$$7 \times 18 = 126 ; 1 + 2 + 6 = 9 \quad \text{☹️}$$

$$7 \times 19 = 133 ; 1 + 3 + 3 = 7 \quad \text{☹️}$$

$$7 \times 20 = 140 ; 1 + 4 + 0 = 5 \quad \text{😊}$$

$$7 \times 21 = 147 ; 1 + 4 + 7 = 12 \quad \text{☹️}$$

$$A = 140 ; B = 115 ; A + B = 255.$$

## Problem 9

$$B = 138 = 6 \times 23 ; 1 + 3 + 8 = 12$$

$$A : 12 \times 8 = 96 < 100 \quad \text{☹️}$$

$$12 \times 9 = 108 ; 1 + 0 + 8 = 9 \quad \text{☹️}$$

$$12 \times 10 = 120 ; 1 + 2 + 0 = 3 \quad \text{😊}$$

$$12 \times 11 = 132 ; 1 + 3 + 2 = 6 \quad \text{😊}$$

$$12 \times 12 = 144 ; 1 + 4 + 4 = 9 \quad \text{☹️}$$

$$12 \times 13 > 150$$

## Problem 9

$$A = 140 ; B = 115 ; A + B = 255 ;$$

$$A = 120 ; B = 138 ; A + B = 258 ;$$



$$A = 132 ; B = 138 ; A + B = 270.$$

## Problem 10

$$\begin{array}{r}
 \phantom{A} B \\
 \times B A \\
 \hline
 \dots\dots B \\
 \dots\dots\dots \\
 \hline
 A X Y B
 \end{array}$$

$B \neq 0$   
 $B \neq A$

×	1	2	3	4	5	6	7	8	9
1		2	3	4	5	6	7	8	9
2	2		6	8	10	12	14	16	18
3	3	6		12	15	18	21	24	27
4	4	8	12		20	24	28	32	36
5	5	10	15	20		30	35	40	45
6	6	12	18	24	30		42	48	54
7	7	14	21	28	35	42		56	63
8	8	16	24	32	40	48	56		72
9	9	18	27	36	45	54	63	72	



Problem 10

$$\begin{array}{r}
 A B \\
 \times B A \\
 \hline
 \dots\dots B \\
 \dots\dots\dots \\
 \hline
 A X Y B
 \end{array}$$

$$62 \times 26 = 1612$$



$$64 \times 46 = 2944$$



$$68 \times 86 = 5848$$



$$35 \times 53 = 1855$$



$$75 \times 57 = 4275$$



$$95 \times 59 = 5605$$



Problem 10

$$A = 1 ; AB < 20$$

$$16 \times 61 = 976 < 1000.$$

$$B = 7 ; 8 ; 9.$$

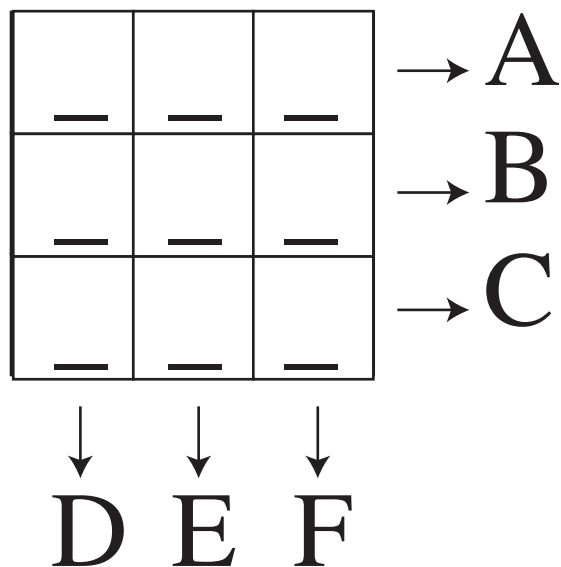
$$17 \times 71 = 1207 ;$$

$$18 \times 81 = 1458 ;$$



$$19 \times 91 = 1729.$$

## Problem 11



$$A + B + C = 45$$

$$D + E + F = 45$$

$$12 + 13 + 14 + 15 + 16 + 17 = 87 ;$$

$$13 + 14 + 15 + 16 + 17 + 18 = 93 ;$$

$$12 + 13 + 14 + 16 + 17 + 18 = 90 ;$$

$$10 + 14 + 15 + 16 + 17 + 18 = 90 ;$$

etc ...

## Problem 11

Example :

3	8	5	→ 16
6	9	2	→ 17
4	1	7	→ 12

↓   ↓   ↓  
13   **18**   14



Problem 12

$$\frac{WROC}{\text{ŁAW}} = \frac{4}{3}$$

$$\text{ŁAW} \leq 987 \longrightarrow WROC \leq 1316$$

$$1000 \times \frac{3}{4} = 750 \longrightarrow \text{ŁA} > 75$$

$$W = 1; R \in \{0; 1; 2; 3\}.$$

$$698 \times \frac{4}{3} < 1000 \longrightarrow \text{Ł} > 6$$

$$\text{Ł} \in \{7; 8; 9\}.$$

Problem 12

$$\frac{WROC}{\overline{LAW}} = \frac{4}{3}$$

$$3 \times \overline{1ROC} = 4 \times \overline{LA1} \longrightarrow C = 8.$$

Problem 12

$$\frac{\overline{WROC}}{\overline{ŁAW}} = \frac{4}{3}$$

$$3 \times \overline{1RO8} = 4 \times \overline{ŁA1} \longrightarrow 0 = 2k$$


$$0 \in \{0 ; 2 ; 4 ; 6\}$$


$$\overline{ŁA1} = 3p \longrightarrow Ł \neq 7$$


$$\overline{ŁA1} \in \{921 ; 951 ; 981\}$$

## Problem 12

$$\frac{\text{WROC}}{\text{ŁAW}} = \frac{4}{3}$$

$921 \times \frac{4}{3} = 1228$  ; 

$951 \times \frac{4}{3} = 1268$  ; 

$981 \times \frac{4}{3} = 1308$ . 

**WROCŁAW = 1268951**



## Problem 13

<i>a</i>	<i>b</i>	6	<i>c</i>	<i>d</i>
<i>e</i>	<i>f</i>	13	7	10
9	8	<i>g</i>	<i>h</i>	<i>i</i>

$6 + 7 + g + f$   
 $= 6 + 7 + 20 + f$   
 $= 33 + f = 4 \times 13$   
 $\rightarrow f = 19$

$6 + 13 + g = 3 \times 13 \rightarrow g = 20$

$10 + 7 + 13 + f + e$   
 $= 10 + 7 + 13 + 19 + e$   
 $= 5 \times 13 \rightarrow e = 16$

$8 + f + b = 8 + 19 + b$   
 $= 3 \times 13 \rightarrow b = 12$

## Problem 13

<i>a</i>	12	6	<i>c</i>	<i>d</i>
16	19	13	7	10
9	8	20	<i>h</i>	<i>i</i>

$9 + 16 + a = 3 \times 13$   
 $\longrightarrow a = 14$

$c + 7 + h = 3 \times 13$   
 $c + h = 32$

$9 + 8 + 20 + h + i = 5 \times 13$   
 $h + i = 28$

$d + 10 + i = 3 \times 13$   
 $\longrightarrow d + i = 29$

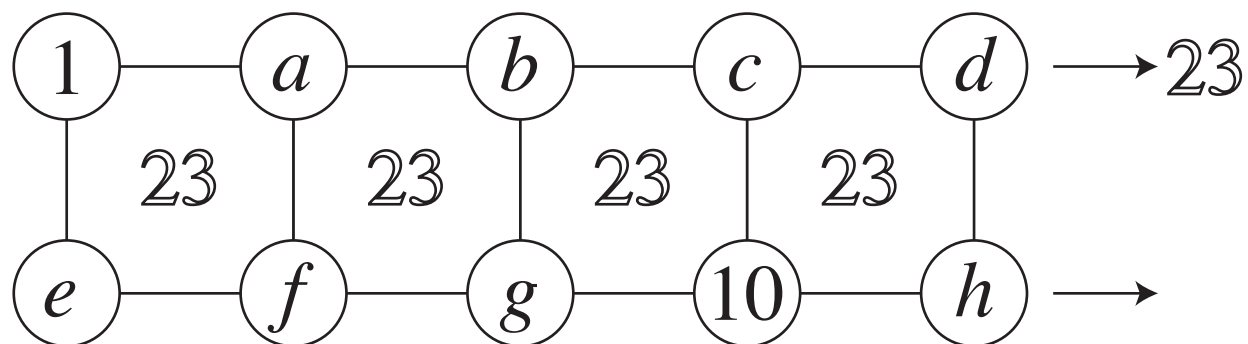
**$C = 15 ; h = 17 ; d = 18 ; i = 11.$**

## Problem 13

14	12	6	15	18
16	19	13	7	10
9	8	20	17	11



## Problem 14



$$1 + e = b + g = d + h ; a + f = c + 10 ;$$

$$1 + e + a + f + b + g + c + 10 = 2 \times 23 = 46 ;$$

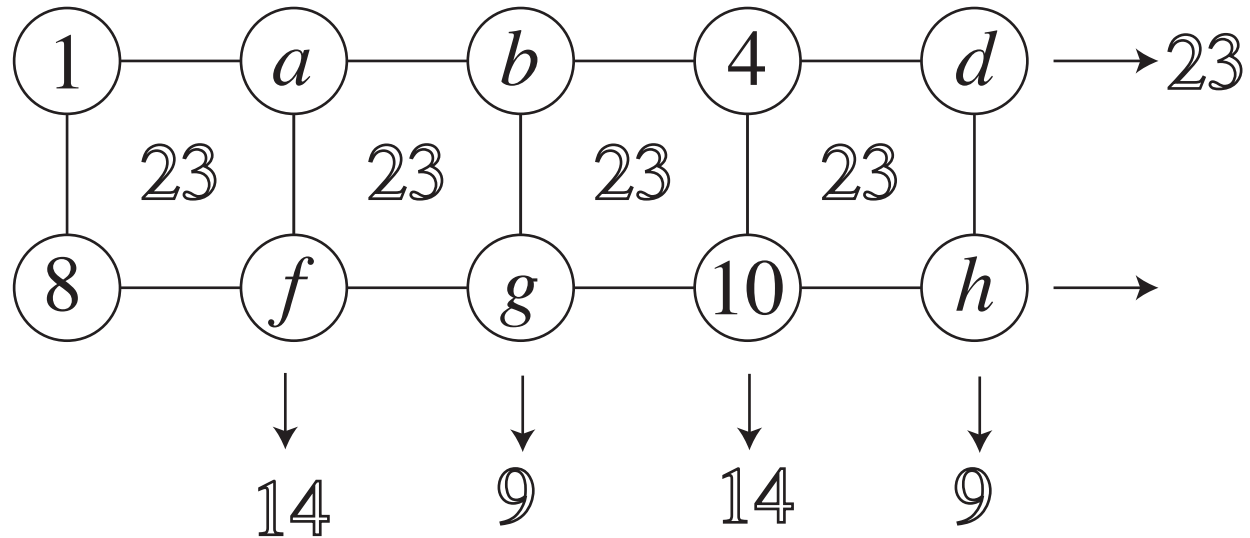
$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55 ;$$

$$\longrightarrow d + h = 55 - 46 = 9 ; \mathbf{e = 8} ;$$

$$a + f + c + 10 = 55 - 3 \times 9 = 28 ;$$

$$\longrightarrow a + f = c + 10 = 14 ; \mathbf{c = 4}.$$

## Problem 14



$$\{2 ; 3 ; 5 ; 6 ; 7 ; 9\}$$

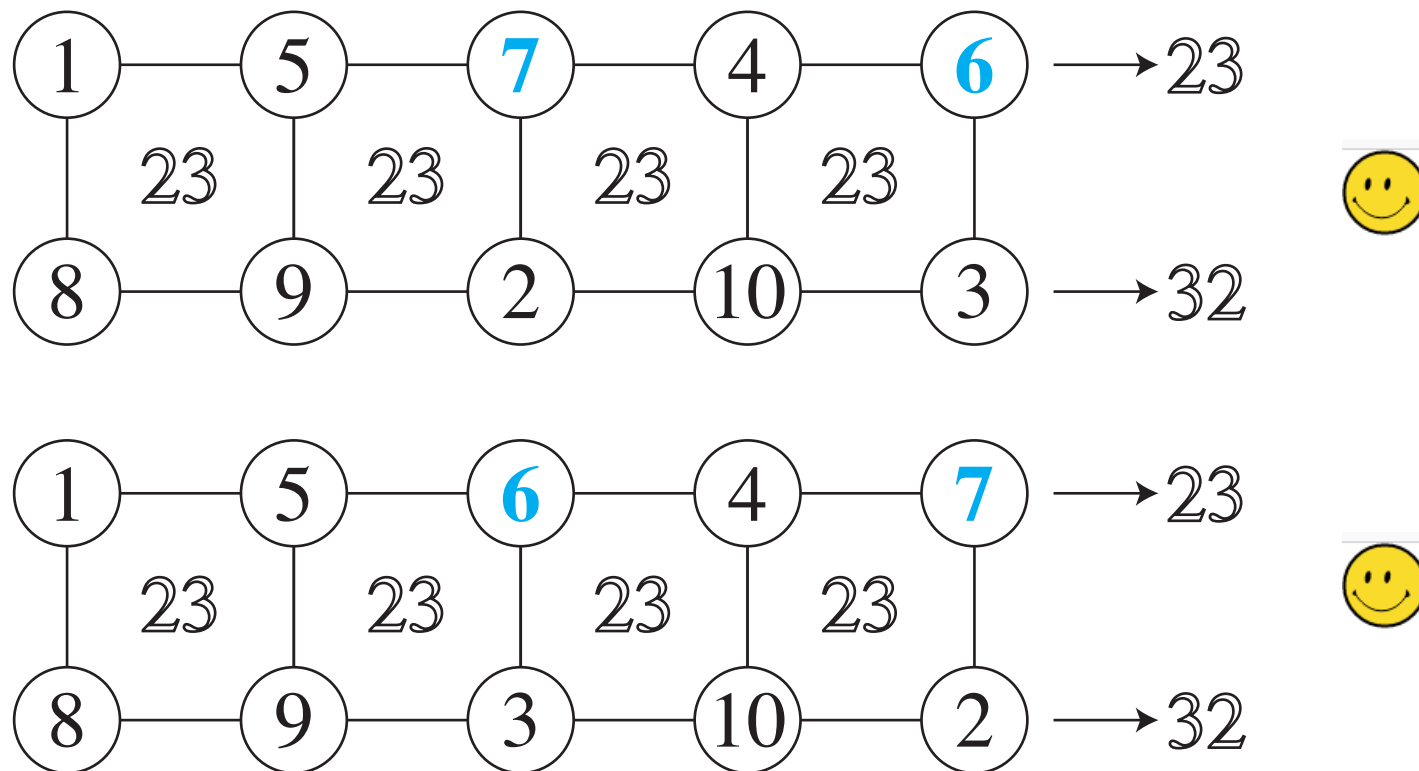
$$\{a ; f\} = \{5 ; 9\} ; \{\{b ; g\} ; \{d ; h\}\} = \{\{2 ; 7\} ; \{3 ; 6\}\}$$

$$1 + a + b + 4 + d = 23 ; \longrightarrow a + b + d = 18$$

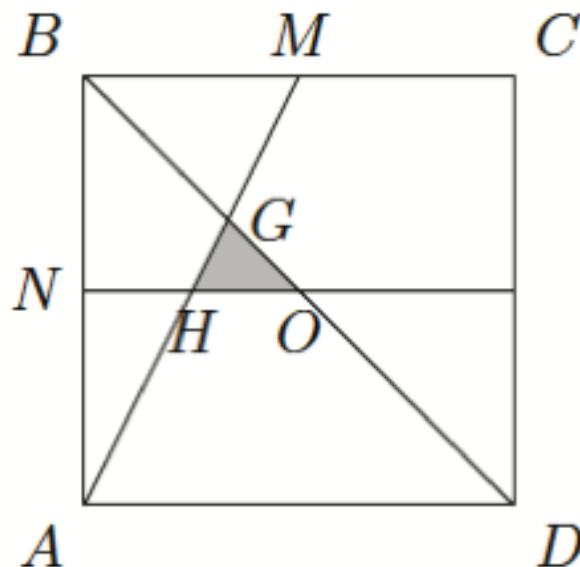
$a = 9$  impossible ;

$$a = 5 \longrightarrow \{b ; d\} = \{6 ; 7\} ; \{g ; h\} = \{2 ; 3\} ;$$

## Problem 14



## Problem 15



$$NO = AD/2 ; HO = NO/2 = AD/4$$

$$\longrightarrow \text{area (HGO)} = \text{area (AGD)}/16.$$

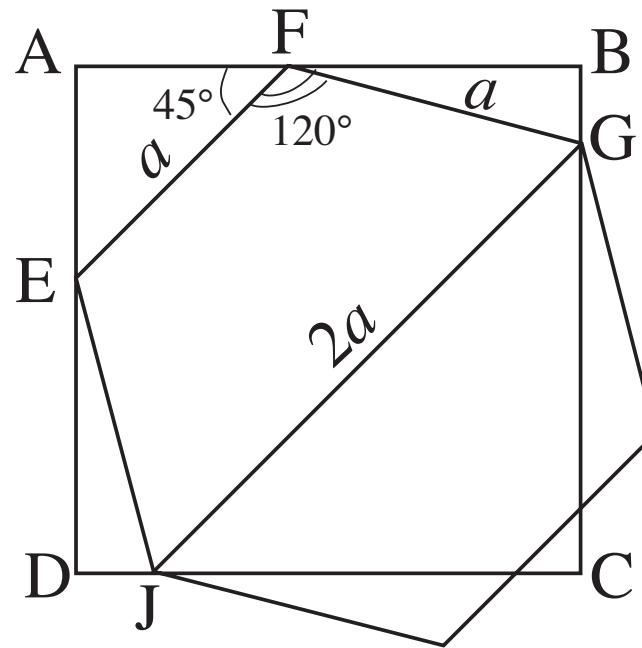
$$GO = GD/4 = OD/3 \longrightarrow BG = 2BO/3 = 2BD/6 ; BD = 3GD/2 ;$$

$$\text{area (ABD)} = 3 \times \text{area (AGD)}/2 ;$$

$$\text{area (ABCD)} = 2 \times 3 \times 16 \times \text{area (HGO)}/2 = \mathbf{480 \text{ m}^2}.$$



## Problem 16



$$EF = FG = a.$$

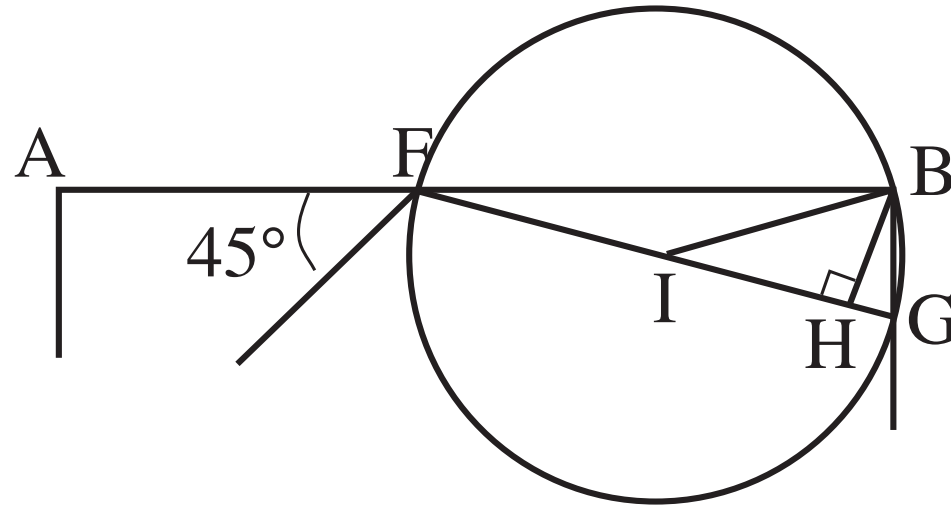
$$\angle AFE = 45^\circ ; \angle EFG = 120^\circ ; \angle GFB = 15^\circ.$$

$$EA = AF = a \sqrt{2}/2 ; \text{area} (AFE) = a^2/4.$$

$$GC = CJ = a\sqrt{2} ; \text{area} (GCJ) = a^2.$$

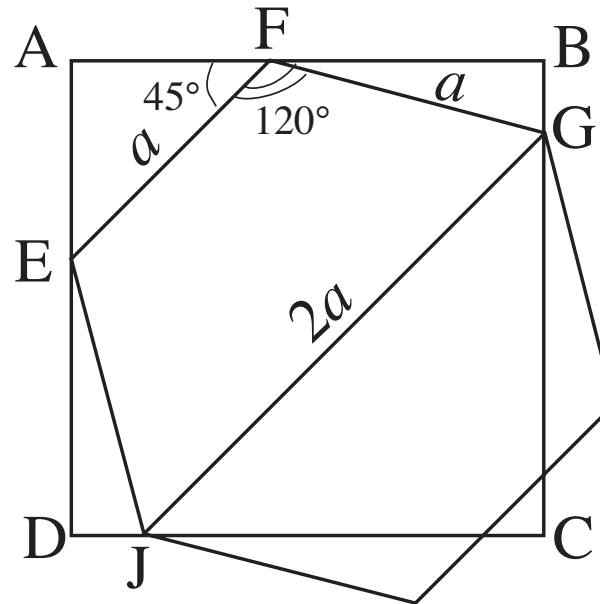


## Problem 16



$IF = IB = IG = a/2$  ;  $\angle BIH = 30^\circ$  ;  
 $(BH) \perp (FG)$  ;  $BH = IB/2 = a/4$ .  
 $\text{area (FBG)} = FG \times BH/2 = a^2/8$ .

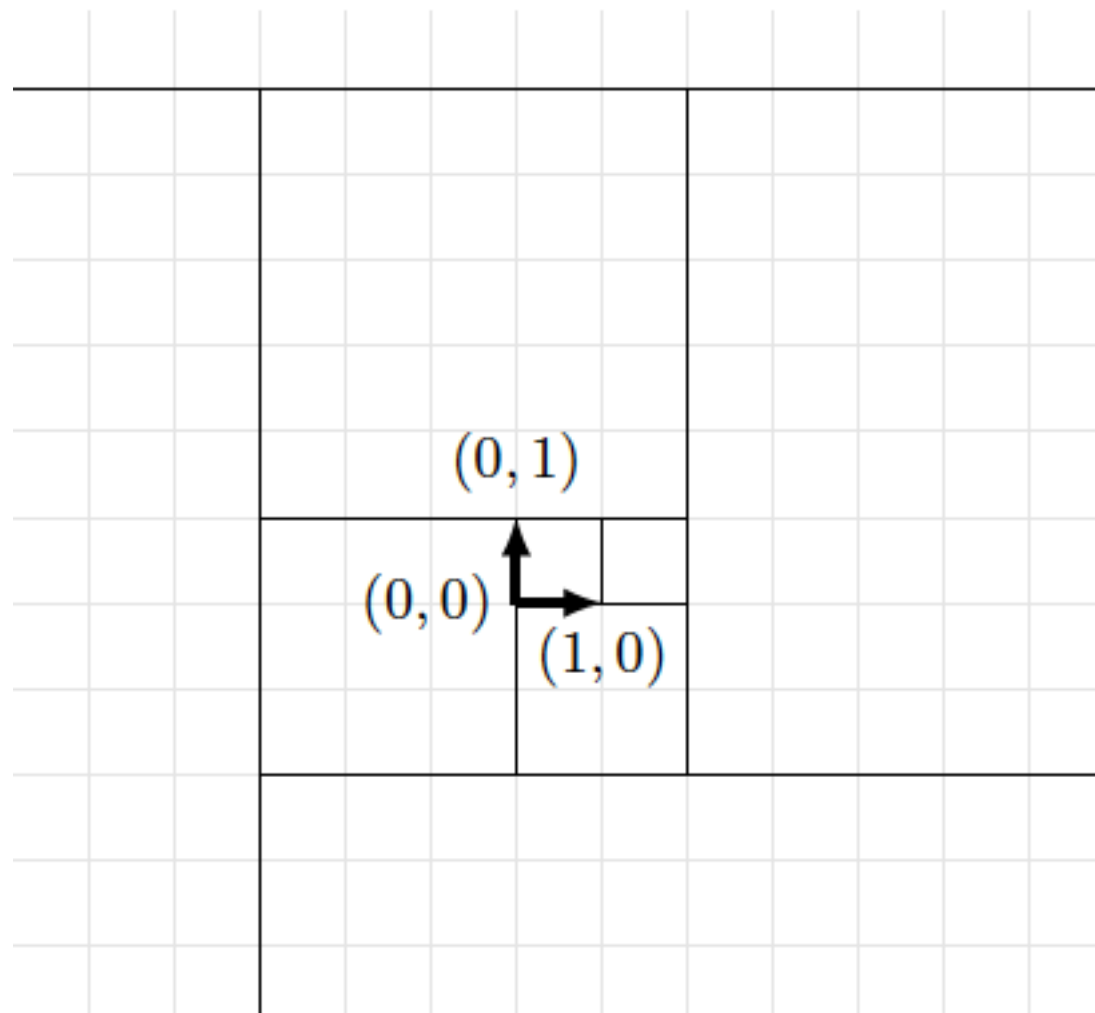
## Problem 16



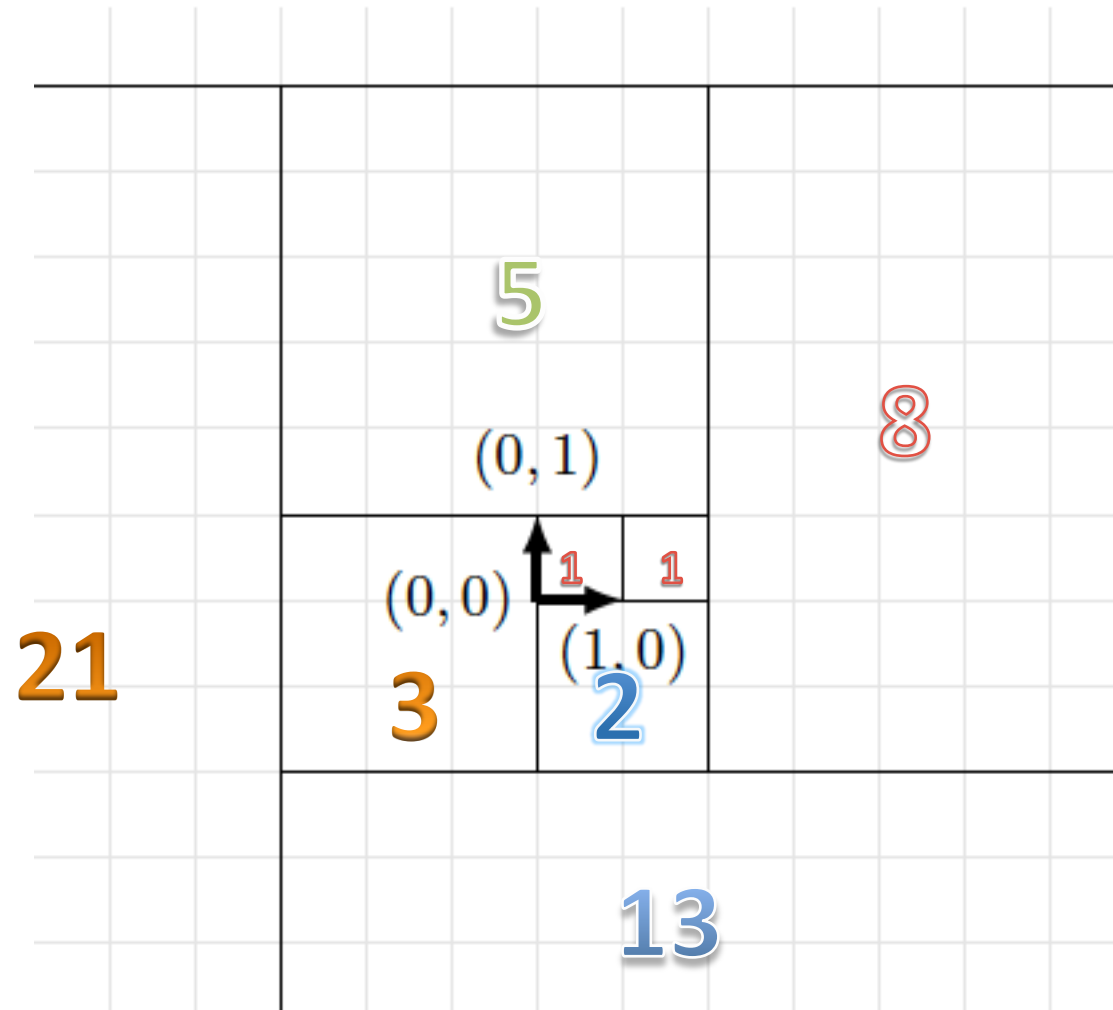
$$\begin{aligned} &\text{Area (AFE)} + \text{area (FBG)} + \text{area (DEJ)} + \text{area (GCJ)} \\ &= a^2/4 + a^2/8 + a^2/8 + a^2 = 3a^2/2 = 216 \text{ cm}^2 ; \\ &\longrightarrow a^2 = 144 \text{ cm}^2 ; a = 12 \text{ cm} ; 2a = \mathbf{24 \text{ cm}}. \end{aligned}$$



## Problem 17



## Problem 17



## Problem 17

Fibonacci sequence :

1 ; 1 ; 2 ; 3 ; 5 ; 8 ; 13 ; 21 ; 34 ; 55 ; 89 ; 144 ;  
 233 ; 377 ; 610 ; 987 ; 1597 ; 2584 ; 4181 ; ...

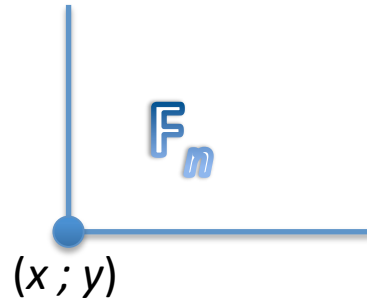
<b>1</b>	<b>5</b>	<b>34</b>	<b>233</b>	<b>1597</b>	<b>10946</b>
1	6	40	273	1870	12816

<b>1</b>	<b>8</b>	<b>55</b>	<b>377</b>	<b>2584</b>	<b>19581</b>
1	9	64	441	3025	22606

<b>3</b>	<b>21</b>	<b>144</b>	<b>987</b>	<b>6765</b>	.....
3	24	165	1152	7917	.....

<b>2</b>	<b>13</b>	<b>89</b>	<b>610</b>	<b>4181</b>	.....
2	15	102	712	4893	.....

## Problem 17



$F_n$	<b>5</b>	<b>34</b>	<b>233</b>	<b>1597</b>	<b>10946</b>
$(x; y)$	$(-3; 1)$	$(-24; 6)$	$(-165; 40)$	$(-1152; 273)$	$(-7917; 1870)$

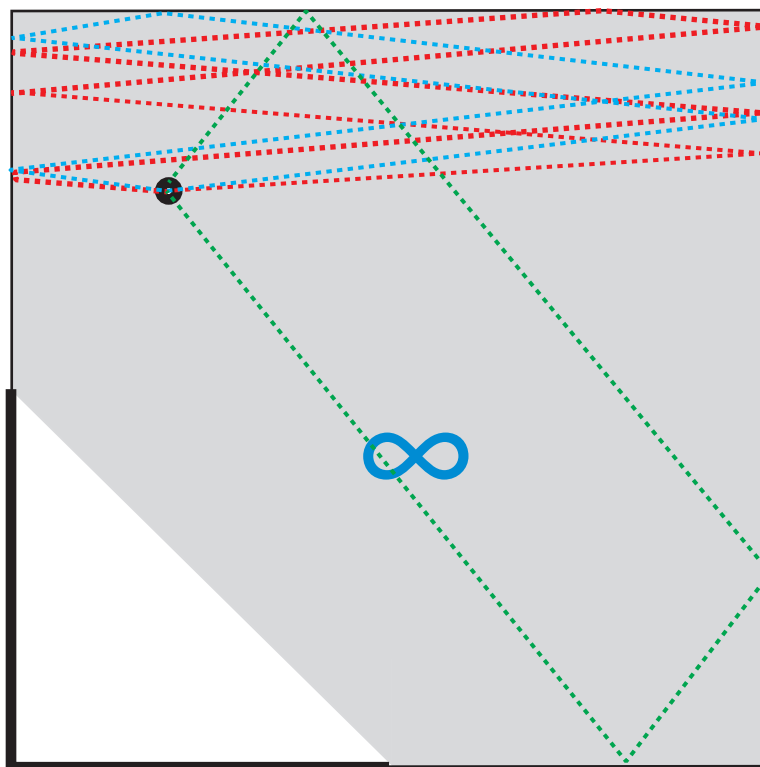
$$-7917 + 10\,946 = 3029; 2023 < 3029;$$

$$1870 + 10946 > 2023.$$

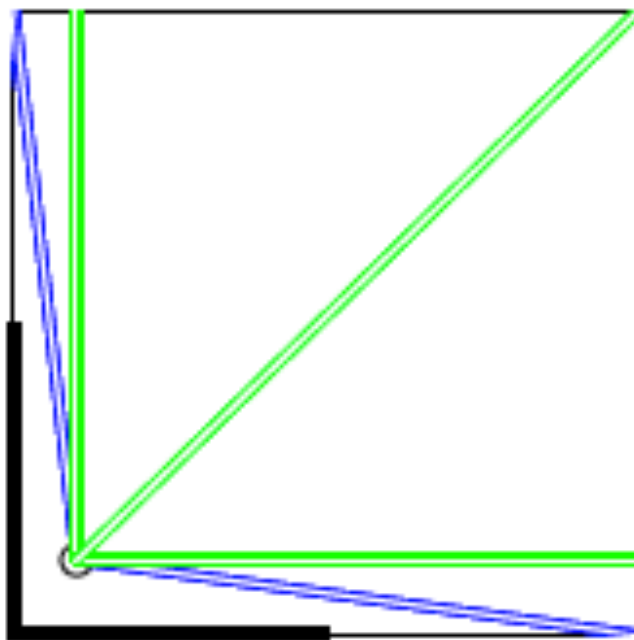
**10946**



## Problem 18



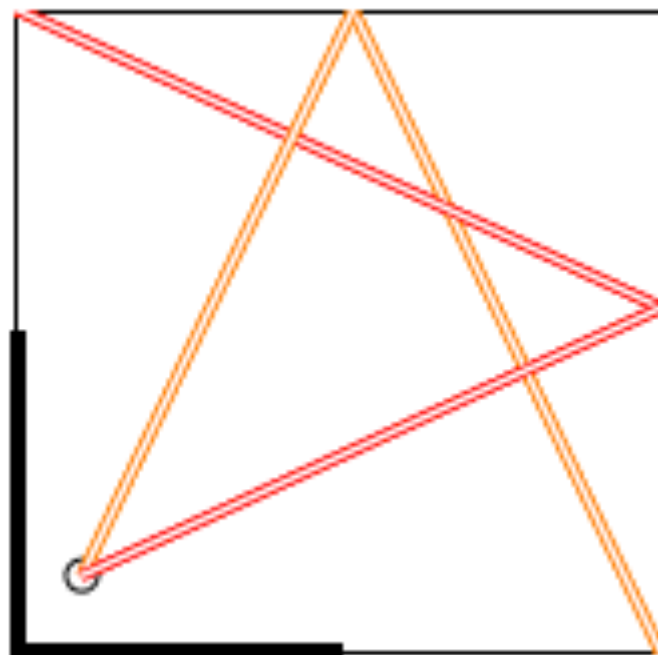
## Problem 18



1 ; 2 ; 3 ; 4 ; 5

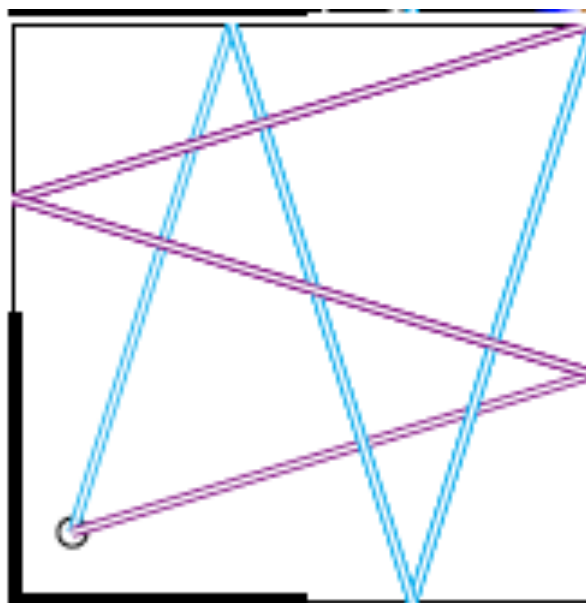


## Problem 18



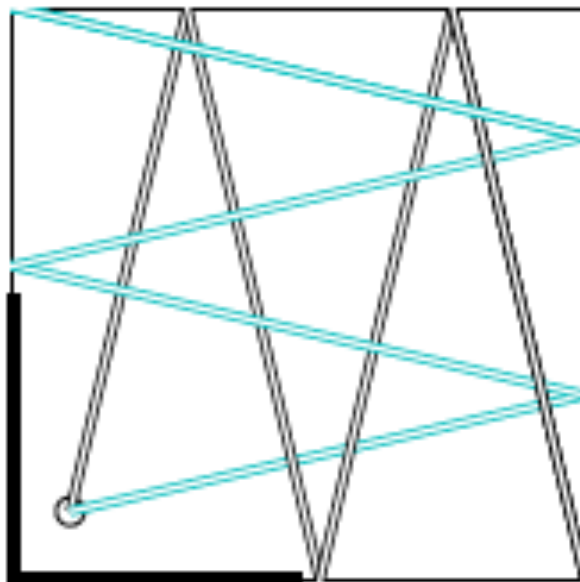
6 ; 7

## Problem 18



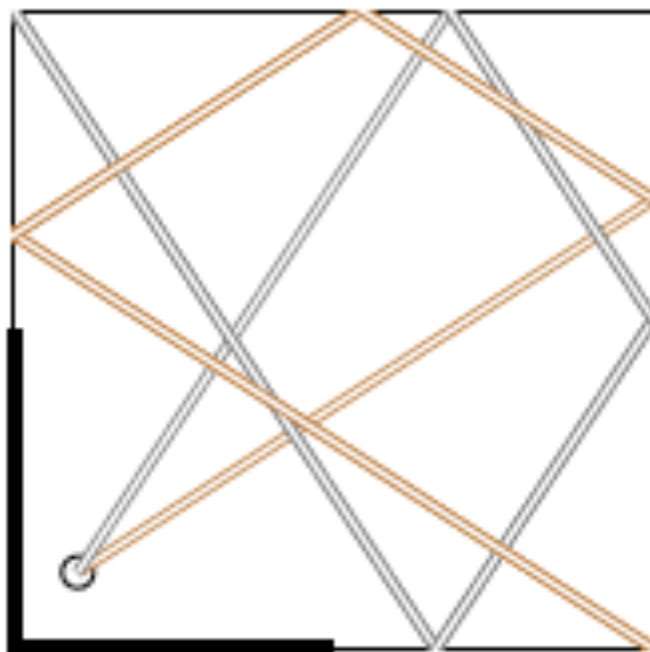
8 ; 9

## Problem 18



10 ; 11

## Problem 18



12 ; **13**

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